CYBERNETICS AND CONTROL THEORY

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Abstract

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ON SOME FEATURES OF MOTION IN AUTOMATIC CONTROL SYSTEMS WITH VARIABLE STRUCTURE POSSESSING A DISCONTINUOUS SWITCHING FUNCTION

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An automatic control system with variable structure is considered, in which the motion is described by the equation

$$P(p^*)(-\psi x_1) = Q(p^*)x_1, \tag{1}$$

where

$$P(p^*) = \sum_{i=0}^{m-1} a_{i+1}p^{*i}, \qquad Q(p^*) = \sum_{i=0}^{n} b_{i+1}p^{*i}, \tag{2}$$

a_i, b_i are constant coefficients, $a_1 \neq 0$, $b_{n+1} = 1$, $n \geq m$; p^* is the operator of the generalized derivative ([1]); x_1 is the controlled coordinate; ψ is a discontinuous function of the system coordinates, with $p\psi = 0$; p is the operator of the ordinary derivative.

In works ([2,3]), in constructing systems with variable structure it was assumed that $P(p^*) = 1$. In such systems the switching function, which determines the instants of discontinuity of ψ, is continuous. If in equation (1) the operator $P(p^*) \neq 1$, then the switching function becomes discontinuous, which leads to the appearance of certain features of the motion in the system.

Instead of equation (1), we shall henceforth proceed from the system of differential equations

$$\frac{dx_i}{dt} = x_{i+1} \qquad (i = 1, 2, \dots, n-1),$$

$$\frac{dx_n}{dt} = -\psi \sum_{i=1}^{m} a_i x_i - \sum_{i=1}^{m} \Delta_i(t) x_i - \sum_{i=1}^{n} b_i x_i, \tag{3}$$

where the function ψ, according to ([3]), has the form

$$\psi = \begin{cases} \omega^2, & \text{for } \sigma x_1 > 0^*, \\ -\omega^2, & \text{for } \sigma x_1 < 0; \end{cases} \qquad \sigma = \sum_{i=1}^{n} c_i x_i; \tag{4}$$

c_i, ω^2 are constants;

$$\Delta_i(t) = \sum_{j=i+1}^{m} a_j C_{j-1}^{i-1} \sum_k h_k \delta^{(j-i-1)}(t - t_k),$$

C_{j-1}^{i-1} is the number of combinations of $j-1$ taken $i-1$ at a time; h_k is the magnitude of the k-th discontinuity of the function $\psi(t)$, i.e. $h_k = \psi(t_k + 0) - \psi(t_k - 0)$; $\delta(t)$ is the Dirac function; t_k is the instant corresponding to the k-th discontinuity, and the summation is carried out over all points of discontinuity.

According to the equations of motion (3), the coordinates $x_{n-m+2}, \dots, x_n$, and consequently also the switching function σ (5), undergo discontinuities.

Introduce new variables $\varphi_1, \varphi_2, \dots, \varphi_n$ such that

$$\varphi_1 = \frac{x_1}{P(p^*)}, \tag{5}$$

* In the case $\sigma x_1 = 0$: $\psi = \omega^2$ for $\sigma x_1 \to +0$, $\psi = -\omega^2$ for $\sigma x_1 \to -0$.

Fig. 1

and the equations of motion take the form

$$\frac{d\varphi_i}{dt} = \varphi_{i+1} \quad (i = 1, 2, \dots, n-1); \qquad \frac{d\varphi_n}{dt} = -\psi \sum_{i=1}^{m} a_i \varphi_i - \sum_{i=1}^{n} b_i \varphi_i; \tag{6}$$

$$\psi = \begin{cases} \omega^2 & \text{for } \sigma \cdot g > 0^*, \\ -\omega^2 & \text{for } \sigma \cdot g < 0; \end{cases} \tag{7}$$

$$\sigma = D(p^*)\varphi_1, \qquad D(p^*) = \sum_{i=0}^{n+m-2} \left(\sum_{i=k+l-2} c_k a_l \right) p^{*i}, \tag{8}$$

$$k = 1, 2, \dots, n; \quad l = 1, 2, \dots, m;$$

$$g = \sum_{i=1}^{m} a_i \varphi_i. \tag{9}$$

From (6) it is obvious that the coordinates $\varphi_1, \varphi_2, \dots, \varphi_n$ are continuous functions of time. Expressions (6), (7), (8) define σ as a linear combination of $\varphi_1, \varphi_2, \dots, \varphi_n$ with coefficients changing discontinuously. Let us denote the switching function by

$$\sigma = \begin{cases} \sigma_1 & \text{for } \psi = \omega^2, \\ \sigma_2 & \text{for } \psi = -\omega^2. \end{cases} \tag{10}$$

As follows from equations (7) and (10), the quantity ψ is determined uniquely only when $\sigma_1 \cdot \sigma_2 > 0$:

$$\begin{aligned} &\psi = \omega^2 \text{ for } g\sigma_1 > 0 \text{ and } g\sigma_2 > 0, \\ &\psi = -\omega^2 \text{ for } g\sigma_1 < 0 \text{ and } g\sigma_2 < 0. \end{aligned} \tag{11}$$

* In the case $\sigma \cdot g = 0$: $\psi = \omega^2$ for $\sigma \cdot g \to +0$, $\psi = -\omega^2$ for $\sigma \cdot g \to -0$.

If the coordinates of the system $\varphi_1, \varphi_2, \dots, \varphi_n$ are such that $g\sigma_1 > 0$ and $g\sigma_2 < 0$, then the quantity ψ may be equal both to ω^2 and to $-\omega^2$. Indeed,

$$\begin{aligned} &\text{for } \psi = \omega^2 \qquad \sigma = \sigma_1 \text{ and } g\sigma > 0, \\ &\text{for } \psi = -\omega^2 \qquad \sigma = \sigma_2 \text{ and } g\sigma < 0. \end{aligned} \tag{12}$$

In the case where the conditions

$$g\sigma_1 < 0, \qquad g\sigma_2 > 0, \tag{13}$$

are simultaneously satisfied,

Fig. 2

there occurs the so-called "sliding mode,"characterized by a discontinuous change of the structure of the system, and the equations of motion take the form

$$\frac{d\varphi_i}{dt} = \varphi_{i+1} \quad (i = 1, 2, \dots, n-1); \qquad \frac{d\varphi_n}{dt} = \overline{\omega} \sum_{i=1}^{m} a_i \varphi_i - \sum_{i=1}^{n} b_i \varphi_i, \tag{14}$$

where

$$\overline{\omega} = \omega^2 \frac{\alpha - 1}{\alpha + 1},$$

and α is a certain continuous function of the system coordinates, equal to the ratio of the times of motion in the sliding mode for $\psi = \omega^2$ and for $\psi = -\omega^2$.

It should be noted that in the automatic-control system under consideration, just as in the system with a continuous switching function ([2,3]), there exists motion in a sliding mode in which one of the quantities σ_1 or σ_2 is equal to zero.

Figure 1 presents an example of a second-order system for which, on the phase plane (φ_1, φ_2), there exist regions I, II, III, where respectively the conditions (11), (11), and (12) are satisfied; in Fig. 2, in regions I, II, III the conditions (11), (11), and (13) are satisfied. The indicated cases are considered for

$$n = 2, \quad m = 2, \quad b_1 = b_2 = 0, \quad a(\varphi_1, \varphi_2) = \text{const}.$$

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REFERENCES

1. I. M. Gel' fand, G. E. Shilov, *Generalized Functions and Operations on Them*, 1959.

2. S. V. Emel' yanov, M. A. Bermant, *DAN*, **145**, No. 4 (1962).

3. S. V. Emel' yanov, V. A. Taran, *Izv. AN SSSR, Energetika i avtomatika*, No. 3 (1962).

Note: Figure translations are in progress. See original paper for figures.