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Abstract

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ON THE POSSIBILITY OF DETECTING AND STUDYING RADIATION BELTS AT LARGE DISTANCES BY METHODS OF RADIO ASTRONOMY

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In the present paper the question is discussed of the possibility of detecting regions with a regular magnetic field at large distances from their magneto-bremsstrahlung (synchrotron or cyclotron) radiation. This radiation in a regular field is linearly polarized ([1]), regardless of whether it falls in the visible part of the spectrum or in the radio range. At the same time, its frequency spectrum and polarization are so characteristic and possess such features that, given the appropriate instruments, it is possible to distinguish this radiation from any other kind and, in particular, from the synchrotron radiation of regions with a chaotic magnetic field. Moreover, if one calculates the characteristics of synchrotron radiation (the Stokes parameters) for fields of various configurations, then from comparison with the observed characteristics one can obtain a number of important data both on the magnetic field itself and on the spatial, energy, and angular distribution of the radiating electrons.

It is simplest to show this for the example of a dipole magnetic field for the following electron distribution function*: $N(E, r_e, \theta, \alpha) = KE^{-\gamma} \exp[-\frac{q^2}{a^2}(r_e - r_0)^2]\, k(n)\, h^{-n/2} \sin^n \alpha$, where $h = (1 + 3\cos^2\theta)^{1/2}/\sin^6\theta$, and $K,\ \gamma,\ q,\ r_0,\ k$ are constants.

Let us assume, for simplicity, that the magnetic moment of the dipole $\mathbf{M}$ is perpendicular to the line of sight, and let us choose a rectangular coordinate system in such a way that the axis oz coincides with the direction of $\mathbf{M}$, and the axis ox with the direction of the line of sight. Then, using the method set forth in ([1]), we obtain the following expressions for the Stokes parameters of the total flux of synchrotron radiation of the radiation belt:

$$I_\nu = B\,\frac{\gamma + 7/3}{\gamma + 1} \int_0^{2\pi} d\varphi \int_{\theta_m}^{\pi-\theta_m} d\theta\, \frac{|f(\theta\varphi)|^{(\gamma+2n+1)/4}}{(1 + 3\cos^2\theta)^{3n/4} (\sin\theta)^{3(\gamma-n)-4}}, \qquad (1)$$

Fig. 1

Figure 1: Fig. 1

$$Q_\nu = B\int_0^{2\pi} d\varphi \int_{\theta_m}^{\pi-\theta_m} d\theta \, \frac{|f(\theta\varphi)|^{(\gamma+2n-3)/4}\left[(3\cos^2\theta-1)^2 - 9\sin^2\theta\cos^2\theta\sin^2\varphi\right]}{(1+3\cos^2\theta)^{3n/4}(\sin\theta)^{3(\gamma-n)-4}}, \tag{2}$$

$$U_\nu = 6B\int_0^{2\pi} d\varphi \int_{\theta_m}^{\pi-\theta_m} d\theta \, \frac{|f(\theta\varphi)|^{(\gamma+2n-3)/4}(3\cos^2\theta-1)\cos\theta\sin\varphi}{(1+3\cos^2\theta)^{3n/4}(\sin\theta)^{3(\gamma-n)-5}}, \tag{3}$$

$$V_\nu = 0,$$

where $B(\gamma, n, M, \nu)$ and $f(\theta\varphi) = (3\cos^2\theta - 1)^2 + 9\sin^2\theta\cos^2\theta\sin^2\varphi$ are expressions common to all parameters (see ([3])), and the quantity θ_m characterizes the latitudinal extent of the radiation belt. The existence of such a boundary follows from the need to take into account the absorption of particles in the denser layers of the atmosphere, as is seen in the example of the Earth' s radiation belt. This boundary is introduced formally** and characterizes the absorption the more accurately the thinner the radiation belt is, i.e., the larger the constant q.

* See ([2]), where this distribution function, in the same notation, is applied to the outer radiation belt of the Earth.

** That is, no account is taken of the change in the distribution function arising from the presence of absorption; see, for example, ([3]).

It follows from expressions (1)–(3) that the frequency dependence of the Stokes parameters is the same, and therefore the degree of polarization and the position angle do not depend on frequency**. The equality to zero of the parameter V means that the radiation is linearly polarized. Owing to the oddness of the integrand in (3), for the total radiation of a symmetric radiation belt the parameter U is also equal to zero, and therefore

$$\rho = \frac{(Q^2+U^2+V^2)^{1/2}}{I} = \frac{Q_\nu}{I_\nu}, \qquad \operatorname{tg} 2\chi = \frac{U}{Q} = 0. \tag{4}$$

From the last equality it follows that either $\chi = 0$ (equatorial polarization), or $\chi = \pi/2$ ("polar" polarization). The choice of one of these values depends on the sign of the parameter Q_ν and, ultimately, is determined by the values of the constants θ^m, γ, and n. It is convenient to distinguish these two cases by the sign of ρ; therefore in the first formula (4) the sign of the modulus has been omitted.

Fig. 2

Figure 2: Fig. 2

Fig. 1

Expressions (1)–(3) determine the Stokes parameters of the total flux of synchrotron radiation in a dipole magnetic field for the assumed distribution function. This function contains 5 constants (K, n, γ, q, r_0), which must be determined, together with the magnitude of the dipole magnetic moment and the parameter θ_m, from the observed values of I_ν, Q_ν, and U_ν. One of these constants, γ, is readily determined from the exponent of the frequency spectrum δ (the spectral index) by means of the relation $\gamma = 2\delta + 1$, well known from the theory of synchrotron radiation ([1]). For the determination of the remaining 4 constants and the quantity M we have only 3 equations, and for a symmetric radiation belt only 2, since in this latter case, as was indicated above, the parameter U_ν is identically equal to zero. Thus, knowledge of the Stokes parameters for the total radiation flux, without the introduction of additional considerations or hypotheses, is insufficient for an unambiguous determination of the characteristics of the radiation belt. This difficulty can be overcome if modern methods of radio-astronomical observations are used. We shall show this with a simple example, when the radio telescope has a pencil-beam pattern and records the radiation of a narrow strip perpendicular to the dipole axis; the Stokes parameters corresponding to this radiation are then equal to the subintegral expressions in formulas (1)–(3), if the integral over φ is evaluated.

In Fig. 1 the radiation characteristics are given as functions of θ for

** This is true provided that the power-law energy spectrum of the electrons with a specified exponent γ covers a sufficiently broad range of energies ([1]).

of the first quadrant $(0 \leqslant \varphi \leqslant \pi,\ 0 \leqslant \theta \leqslant \pi/2)$ and of the particular case $\gamma = 3,\ n = 0$, when the integrals over φ are simply expressed in terms of elementary functions. The value of the coefficient B (see (3)) is taken as the unit along the ordinate axis. For the indicated values of γ and u, as is seen from the figure, the spectral density of the radiation I_ν rapidly decreases with increasing θ, which simply reflects the fact that the strength of the dipole magnetic field H decreases as the polar angle θ increases. The position angle, changing smoothly from 0 to π, at $\theta \simeq 54°40'$ is equal to $\pi/2$, while the degree of polarization is practically constant and equal to its value in a homogeneous field (4): $\rho = \rho_0 = (\gamma + 1)/(\gamma + 7/3) = 0.75$. In the same figure the curve ρ_p is given for the radiation of the entire ring $(0 \leqslant \varphi \leqslant 2\pi)$ under symmetry of the radiation belt. It is seen from the figure that ρ_p varies from zero to ρ_0, and in the angular interval $35° \leqslant \theta \leqslant 70°$ Q_ν and ρ_p are negative and the corresponding portion of the ρ_p curve characterizes "polar" polarization; outside this interval the polarization is equatorial.

Fig. 2

Fig. 3

Figure 3: Fig. 3

In interpreting observational results it is convenient to use the dependence of the radiation characteristics on the coordinate $z/r_0 = \sin^2\theta\cos\theta$; the corresponding graphs $I_\nu(z/r_0)$ and $\rho_\mathrm{p}(z/r_0)$ for the case $\gamma = 3,\ n = 0$ are shown in Fig. 2. When the emitting region passes through the main lobe of the radio telescope, the first contact will occur at the value $\theta = \theta_k \simeq 55^\circ$, when $z \simeq 0.385 r_0$, ρ_p is negative, and the polarization is "polar." For arbitrary z, if the radiation belt is sufficiently "thin," the total radiation of two rings will be recorded (curves I and ρ_p in Fig. 2), corresponding to two different values of θ, indicated on the upper axis. However, in the particular case under consideration, $\gamma = 3,\ n = 0$, the spectral density of the radiation I_ν falls so rapidly with increasing θ that in practice it is necessary to take into account only the radiation of the inner ring, for which $\theta < \theta_k$ (curve I_2). As z decreases, I_ν increases all the time, while the polarization, decreasing, remains "polar" down to the value $z = z_0 = 0.27 r_0$, at which $\theta \simeq 35^\circ$ and $\rho_\mathrm{p} = 0$. If the latitudinal boundary of the radiation belt is $\theta_m < 35^\circ$, then with a further decrease of z the increase of I_ν will continue, but the direction of the predominant oscillations of the electric vector will change by $\pi/2$, i.e., the polarization will become equatorial. Upon reaching $\theta = \theta_m$ the intensity will drop sharply to the value corresponding to the radiation of the outer ring (curve

I_1 in Fig. 2). For negative values of z the picture is repeated, but in reverse order; therefore, in the case under consideration $\gamma = 3$, $n = 0$ the source in the z direction is double. It is easy to show that the source is also double in the y direction.

Experimental determination of the position of the intensity maximum thus makes it possible, in some cases, to determine one of the unknown parameters θ_m even without recourse to polarization observations. On the other hand, the value $z = z_0$ at which the degree of polarization is minimal and a rotation of the plane of polarization by 90° occurs depends, for known γ, only on the parameter n. If the corresponding graphs are constructed for various values of n, then this parameter can be determined from the experimentally found value of z_0.

Fig. 3

In Fig. 3 the curves $I_\nu(z/r_0)$ and $\rho(z/r_0)$ are shown for another particular case, when $\gamma = 1$ and $n = 1$. For these parameter values the spectral radiation density I_ν increases with increasing θ, and therefore the main contribution to I_ν is made by the outer ring, for which $\theta > \theta_k$ (curve I_1 in Fig. 3 is situated above curve I_2), and the extent of the emitting region in the equatorial direction is $l_e \simeq 2r_0$. If one takes as the natural boundary of the emitting region $\theta = \theta_c$, at which I_ν is e times smaller than its value in the equatorial plane, then the ratio of the polar extent to the equatorial one is

$$l_p/l_e = \sin^2\theta_c \cos\theta_c \leqslant 0.385,$$

where the equality sign applies to the case when $\theta_c \lesssim 55^\circ$. With the experimentally known values of l_p and l_e, θ_c is found from this, and, for known γ, the value of the parameter n is found with the aid of the graph. It can be shown that an increase of the spectral density I_ν with increasing θ is possible only for $n > \gamma - \frac{4}{3}$, and practically only in this case is the inequality $l_p/l_e \leqslant 0.385$ possible.

In many practically important cases the orientation of the dipole moment in space will be unknown, and a more perfect observational technique is necessary, one making it possible to obtain the distribution of the radiation characteristics in the image plane.

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Note: Figure translations are in progress. See original paper for figures.