GEOPHYSICS

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1963

SovietRxiv

Abstract

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GEOPHYSICS

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CALCULATION OF THE PARTICLE SPECTRUM FROM DATA ON SPECTRAL TRANSPARENCY

(Presented by Academician A. A. Lebedev, February 25, 1963)

1. The development of methods for calculating the particle spectrum from information contained in scattered light is one of the most urgent problems in the optics of turbid media.

In the case of "dilute" systems, when one may restrict oneself to consideration of single scattering, the problem reduces to the inversion of a Fredholm integral equation of the first kind

$$\varphi(y) = \int_0^\infty F(x, y) f(x)\, dx. \tag{1}$$

Here $f(x)$ is the particle distribution function with respect to size; $F(x, y)$ is the kernel of the equation, known from the theory of light scattering by a particle; $\varphi(y)$ is an experimentally determined function.

Inverting (1) by directly replacing it with an algebraic system leads to fundamental difficulties characteristic of Fredholm integral equations of the first kind ([3]). The resulting system proves to be ill-conditioned—inevitable small errors in the determination of $\varphi(y)$ and $F(x, y)$ lead to enormous errors in $f(x)$. Thus, a formally correct solution of the corresponding algebraic system gives a physically meaningless result ([2], [4]).

In ([1]) an inversion formula for equation (1) is given for the case in which $\varphi(y)$ denotes the spectral transparency of the system. However, because of the above-noted poor stability of inversion problems, a direct calculation by the formula indicated in ([1]) proves practically impossible. Calculations become possible only after eliminating, in a general form, the causes producing instability of the solution with respect to small errors of measurement and rounding. This has been accomplished in the present work. As a result, an inversion scheme has been found that contains moderate requirements on the accuracy of measurements. We note that this scheme makes it possible to calculate the particle spectrum of a system solely from data on its transparency, without making any additional assumptions about the character of the spectrum sought.

2. In accordance with ([1]), let us introduce the Mellin transform $2L(p)$ of the monodisperse (theoretical) scattering cross section $2K(\delta)$ and the Mellin transform $G(p)$ of the dimensionless polydisperse (experimental) scattering coefficient $g(\nu^*) = g^*(\nu^*) r_0$, where

$$\nu^* = \frac{1}{\lambda}, \qquad \delta = \beta \frac{r}{\lambda}, \qquad \beta = 2\pi(m-1), \qquad r = a r_0, \qquad \nu^* = \nu r_0^{-1}. \tag{2}$$

Here λ is the wavelength, r is the particle radius, m is the refractive index, r_0 is a length scale, and a is the dimensionless particle radius. For the particle distribution function with respect to sizes $f(r)$ we have

$$m(a) = \frac{1}{2\pi i} \int\limits_{c-i\infty}^{c+i\infty} \frac{G(1-p)}{L(1-p)} a^{-p}\, dp, \qquad f(r) = \frac{m(a)}{2\pi a^2} \frac{1}{r_0^4}. \tag{3}$$

Computing the Mellin transform $K(\delta)$, we find

$$L(p) = -\frac{2^{1-p} \cos\frac{\pi p}{2}\, \Gamma(p)}{\beta^p (2-p)} \qquad (-2 < \operatorname{Re} p < 0). \tag{4}$$

Substituting (4) into (3), we obtain the fundamental equation of the problem

$$m(a) = -\frac{2\beta}{\pi} \left\{ \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} (1+p)\Gamma(p) \cos\frac{\pi p}{2} (2\beta)^{-p} \left[\int_0^\infty g(\nu_\beta) \nu^{-p} d\nu \right] a^{-p}\, da \right\}$$

$$(-2 < c < 0). \tag{5}$$

3. The fundamental equation can be used both when the transparency is specified analytically and when it is specified in tabular (graphical) form. An analytical expression for the polydisperse scattering coefficient may be either the result of theoretical analysis or of smoothing experimental data.

Of greatest interest is the case of tabular specification of the polydisperse scattering coefficient, on which we shall dwell. Let, in the interval $0 < \nu^* < \tau^*$, all distinct maxima $g^*(\nu^*)$ be contained, and let the values $g^*(\nu_j^*)$ $(j = 1, \dots, n)$ be measured at points ν_j^* of this interval (including on the decreasing portion of the curve g^*). We introduce the dimensionless quantities

$$x = 2\beta \nu^* r_0, \qquad \tau = 2\beta \tau^* r_0, \qquad g\left(\frac{x}{2}\right) = r_0 g^*(\nu^*). \tag{6}$$

Then the following relation holds:

$$f(r) \simeq \frac{-1}{2\pi^2 a^2 r_0^4} \left[\sum_{j=1}^{n} g\left(\frac{x_j}{2}\right) \omega(a x_j) \Delta x_j + c_0 \tau \omega_0(a\tau) + c_2 \frac{\omega_2(a\tau)}{\tau} \right]. \quad (7)$$

Here

$$\omega(y) = y \sin y + \cos y - 1; \qquad \omega_0(y) = \cos y - 2\frac{\sin y}{y} + 1;$$

$$\omega_2(y) = \cos y - 1, \quad (8)$$

and we have compiled detailed tables of these functions. The values c_0 and c_2 are determined with the aid of an empirical formula of the form

$$g\left(\frac{x}{2}\right) = c_0 + \frac{c_2}{x^2} \quad (9)$$

by processing the transparency data on the decreasing portion of this curve.

4. Exact inversion of equation (1) has made it possible to eliminate the source of gross errors that arise in the approximate solution of Fredholm equations of the first kind. In the scheme of Sec. 3 there occur only insignificant errors associated with replacing the integral by a quadrature formula (already in the final result). Calculations for numerous examples show that, when the accuracy of the transparency measurement is 1%, the error of the spectrum calculated by the working formula (7) is of the order of 5%.

5. The wavelength range in which transparency data are required is determined by the formulas

$$\lambda_{\text{min}} \simeq \beta r_{\text{m}}, \qquad \lambda_{\text{max}} \simeq 2.5\, \beta r_{\text{m}}, \quad (10)$$

where r_{m} is the mode of the distribution sought.

Thus, for example, for particles of atmospheric aerosol with $r_{\text{m}} = 0.1\mu$, transparency measurements should be performed in the region from 0.21 to 0.52μ, and for fog droplets with $r_{\text{m}} = 1\mu$, from 2.1 to 5.2μ. In these estimates the refractive index was taken to be 1.33.

Main Geophysical Observatory
named after A. I. Voeikov

Received
22 II 1963

CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.