PHYSICS

I. B. LEVINSON, S. S. FEL, P. Sh. FRIDBERG

1963

SovietRxiv

Abstract

Full Text

PHYSICS

I. B. LEVINSON, S. S. FEL, P. Sh. FRIDBERG

INTEGRAL EQUATION FOR THE APERTURE FIELD IN ELECTROMAGNETIC COUPLING OF TWO VOLUMES

(Presented by Academician B. A. Vvedenskii, 28 VI 1963)

The applicability of the currently existing methods for calculating the electromagnetic coupling of waveguides and resonators is limited by various restrictive conditions that prevent the use of these methods in a number of practical problems. In the works of Ya. N. Feld ([1]), the problem of radiation from a waveguide into free space is considered (i.e., the coupling of the internal and external regions of a waveguide); however, the integrodifferential equation obtained is valid only for a narrow slot, when $d/\lambda \ll 1$, where d is the slot width and λ is the wavelength. Methods for solving this equation have been developed only for an exponentially narrow slot, when $|\ln d/\lambda|^{-1} \ll 1$, which is a very severe restriction. For the applicability of G. Bethe' s method ([2]), the ordinary (not exponential) smallness of the dimensions of the coupling aperture is sufficient, but it must hold for the dimensions in all directions. In addition, for this method to be applicable, the aperture must lie far from regions of sharply varying surface curvature or discontinuities (for example, from the edge of a waveguide), since the formulas apply directly only to the coupling of two half-spaces separated by a plane with an aperture. Because of all these restrictions, many essential details of the phenomenon are missed in such calculation methods. For example, for Bethe' s small aperture and Feld' s exponentially narrow slot, the field at the coupling aperture is completely independent of the geometry of the system, i.e., the reaction of the load is not taken into account.

In the present work, a general vector integral equation is formulated for the field $\mathbf{E}_a$ at the aperture in the case of coupling of two volumes (finite or infinite) excited by sources or by fields incident from infinity.

No restrictions are imposed on the geometry of the system or on the size of the coupling aperture. We use the same method that was used by H. Levine and J. Schwinger ([3]) in considering the problem of coupling two half-spaces through an aperture of arbitrary shape and size in an infinite plane screen. This method consists in the use of affine Green' s functions for the vector field.

Consider two volumes bounded by ideally conducting walls and filled, in general, with different media. The volumes are connected by a common aperture; if the volumes are filled with identical media, then any surface may be chosen as the common surface, while if the media are different, the shape of the common coupling surface is determined by the boundary between the media. The Green' s functions of both volumes are assumed known in the absence of coupling, i.e., when the aperture surface is "metallized."

These functions satisfy the following equation

$$\left(\vec{\nabla}^{\,2} + k_0^2\right) \mathbf{G}(\mathbf{r}, \mathbf{r}') = -\mathbf{I}\delta(\mathbf{r} - \mathbf{r}'), \tag{1}$$

where $k_0 = \omega\sqrt{\varepsilon\mu}$ is the dispersion law for waves in the filling medium, and $\mathbf{I}$ is the unit affine tensor. In addition, on the surface of the walls S bounding

the corresponding volume, the boundary conditions are satisfied

$$\mathbf{n} \times \mathbf{G}(\mathbf{r}, \mathbf{r}') = 0 \quad \text{for } \mathbf{r} \text{ on } S, \tag{2}$$

$$\operatorname{div} \mathbf{G}(\mathbf{r}, \mathbf{r}') = 0 \quad \text{for } \mathbf{r} \text{ on } S, \tag{3}$$

where $\mathbf{n}$ is the unit vector of the outward normal to the surface at the point $\mathbf{r}$. If the volume is unbounded, then at infinity the Green' s functions must contain only outgoing waves. In (3) solenoidal Green' s functions are used, satisfying condition (3) in all space; however, such functions are unsuitable for solving problems with excitation by charges and, moreover, finding them in a number of cases is difficult.

The field in each volume can be represented as the sum of the field $\mathbf{E}^0, \mathbf{H}^0$, which would be excited in the absence of the aperture, and the field $\mathbf{E}', \mathbf{H}'$ scattered by the aperture:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^0(\mathbf{r}) + \mathbf{E}'(\mathbf{r}), \qquad \mathbf{H}(\mathbf{r}) = \mathbf{H}^0(\mathbf{r}) + \mathbf{H}'(\mathbf{r}). \tag{4}$$

Using the known vectors of Green' s theorem [4], one can express the scattered field through the tangential component of the aperture field

$$\mathbf{E}'(\mathbf{r}) = -\int ds'\, \mathbf{n} \times \mathbf{E}_a(\mathbf{r}') \cdot \operatorname{rot}' \mathbf{G}(\mathbf{r}, \mathbf{r}'). \tag{5}$$

Here $\mathbf{n}'$ is the vector of the outward normal at the point $\mathbf{r}'$, and the integration is carried out over the surface common to both volumes. The equation for the field $\mathbf{E}_a$ is obtained from the condition of matching the tangential components of the total magnetic field $\mathbf{H} \times \mathbf{n}$ when approaching the aperture from the side of each of the volumes. Therefore, using (5), we compute the tangential component of the scattered magnetic field

$$\mathbf{H}'(\mathbf{r}) \times \mathbf{n} = -\int ds'\, \vec{\eta}(\mathbf{r},\mathbf{r}') \cdot \mathbf{E}_a(\mathbf{r}'). \tag{6}$$

Here

$$\vec{\eta}(\mathbf{r},\mathbf{r}') = i\frac{\eta_0}{k_0}\,\mathbf{n} \times \left(\vec{\nabla} \times \mathbf{G}(\mathbf{r},\mathbf{r}') \times \vec{\nabla}'\right) \times \mathbf{n}', \tag{7}$$

$\eta_0 = \sqrt{\varepsilon/\mu}$ is the wave admittance of the medium filling the corresponding volume, and $\vec{\nabla}'$ acts on $\mathbf{G}$ and does not act on $\mathbf{n}'$.

Let us now write equations of type (6) for the scattered fields $\mathbf{H}_1'$ and $\mathbf{H}_2'$; in doing so, $\vec{\eta}$ is replaced by $\vec{\eta}_1$ and $\vec{\eta}_2$, which will be calculated from (7) with the corresponding Green' s functions $\mathbf{G}_1$ and $\mathbf{G}_2$, normals $\mathbf{n}_1$ and $\mathbf{n}_2$, and medium parameters $\eta_{0,1}/k_{0,1}$ and $\eta_{0,2}/k_{0,2}$. The field $\mathbf{E}_a$ may be taken to be the same in computing $\mathbf{H}_1'$ and $\mathbf{H}_2'$, since in fact only the tangential component of this field enters (6), and it is continuous on the interface between the media.

The matching condition now leads to a vector integral equation for the field at the aperture

$$\int ds'\, \mathbf{N}(\mathbf{r},\mathbf{r}') \cdot \mathbf{E}_a(\mathbf{r}') = \mathbf{K}^0(\mathbf{r}). \tag{8}$$

The kernel and the right-hand side of this equation each contain two terms corresponding to the coupled volumes:

$$\mathbf{N}(\mathbf{r},\mathbf{r}') = \vec{\eta}_1(\mathbf{r},\mathbf{r}') + \vec{\eta}_2(\mathbf{r},\mathbf{r}'), \tag{9}$$

$$\mathbf{K}^0(\mathbf{r}) = \mathbf{n}_1 \times \mathbf{H}_1^0(\mathbf{r}) + \mathbf{n}_2 \times \mathbf{H}_2^0(\mathbf{r}). \tag{10}$$

Let us now clarify the physical meaning of equation (8). Obviously, (10) is the current induced by the exciting sources on the "metallized" aperture. The meaning of the quantity $\vec{\eta}$ can be understood in the following way. Let us suppose···

an aperture has been made in the surface of a certain volume, and this aperture is loaded by another volume. We wish to take the presence of the aperture and of the load into account by means of effective boundary conditions for the first volume, analogously to the way this is done with Leontovich boundary conditions when the load is an absorbing wall. For this purpose we use (6) for the load volume; moreover, since there are no sources there, $\mathbf{H}' = \mathbf{H}$. We shall now bring the points $\mathbf{r}$ closer to the aperture; by virtue of the continuity of the tangential components, $\mathbf{H}$ may be regarded as the field in the first volume, and

$\mathbf{E}_a$ may be replaced by $\mathbf{E}$ in the first volume near the aperture. Then we obtain the boundary condition for the fields in the first volume

$$\mathbf{H}(\mathbf{r}) \times \mathbf{n} = \int ds' \, \vec{\eta}(\mathbf{r}, \mathbf{r}') \, \mathbf{E}(\mathbf{r}'), \tag{11}$$

where $\vec{\eta}$ refers to the load volume and $\mathbf{n}$ is the outward normal to the volume under study. This boundary condition is analogous to the Leontovich condition

$$\mathbf{H}(\mathbf{r}) \times \mathbf{n} = \eta_0 \mathbf{E}(\mathbf{r}), \tag{12}$$

but differs from it in its nonlocal character. Therefore $\vec{\eta}(\mathbf{r}, \mathbf{r}')$ may be understood as the input admittance of the volume through the aperture. This is also seen from (6), if $\mathbf{E}_a$ is regarded as an emf applied to the edges of the aperture, and $\mathbf{H}' \times \mathbf{n}$ as the "current" excited in the volume.

If there are excitation sources in the volume and it is considered as a generator, then $\vec{\eta}(\mathbf{r}, \mathbf{r}')$ will be the internal admittance of the generator. Therefore (8) is simply the "complete Ohm' s law" for two current generators with connected terminals. The field $\mathbf{E}_a$, analogously to the potential difference at the terminals, and the currents (10), induced by the exciting sources, play the role of the electromotive force of the generators.

From equation (8) one can obtain an equation for a slot. To do this, one should neglect the longitudinal (along the slot) component of the field $\mathbf{E}_a$ and assume that the dependence of the transverse component of $\mathbf{E}_a$ on the coordinate varying across the slot is determined by the electrostatic problem for two half-planes separated by a gap. The equation obtained is equivalent to Feld's integro-differential equation (1), but is an integral equation, since the singular part of the Green' s function has not been separated out.

When computing the integral (6) and bringing the points $\mathbf{r}$ closer to the surface, a well-known precaution must be observed. If expansions in eigenfunctions in the form of infinite series-integrals are used for $\mathbf{G}$, then divergent expressions will be obtained for η. Therefore, an exponential convergence factor with a small parameter Δ (of the dimension of length) should be introduced into the series-integral for $\mathbf{G}$, which corresponds to smearing the δ-function over a volume Δ^3. Such a limiting δ-function may be used in (6) if the distance h from the point $\mathbf{r}$ to the surface of the aperture is much greater than Δ. Therefore, after first computing (6) for $h > 0$, one should then carry out the following limiting transition: first $\Delta \to 0$, and then $h \to 0$.

The authors consider it their pleasant duty to express gratitude to A. A. Pistol' kors, Ya. N. Feld, L. D. Bakhrakh, and B. Z. Katsenelenbaum, as well as to the participants of the seminars led by them, for fruitful discussion of the work.

Received
28 VI 1963

CITED LITERATURE

1. Ya. N. Feld, *Fundamentals of the Theory of Slot Antennas*, Moscow, 1948.

2. H. Bethe, Phys. Rev., **66**, 163 (1944).

3. H. Levine, J. Schwinger, Comm. Pure and Appl. Math., **3**, 355 (1950).

4. F. M. Morse, G. Feshbach, *Methods of Theoretical Physics*, IL, 1960.

Note: Figure translations are in progress. See original paper for figures.