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V. V. DUDUKALENKO, D. D. IVLEV

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Abstract

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THEORY OF ELASTICITY

V. V. DUDUKALENKO, D. D. IVLEV

ON THE COMPRESSION OF A STRIP OF HARDENING PLASTIC MATERIAL BY RIGID ROUGH PLATES

(Presented by Academician Yu. N. Rabotnov, 12 III 1963)

The paper considers the linearized relations of the theory of plane strain of an anisotropically hardening material ([1–5]) for the case of small deformations, on the basis of which a generalization is given of Prandtl' s solution ([6,7]) for the compression of a strip by rigid rough plates.

The fundamental relations of the theory of plane strain of an anisotropically hardening rigid-plastic material may be written in the form

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \qquad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0; \tag{1}$$

$$\left[(\sigma_x - \sigma_y) - c(\varepsilon_x - \varepsilon_y)\right]^2 + 4(\tau_{xy} - c\varepsilon_{xy})^2 = 4k^2, \qquad k = \text{const}, \quad c = \text{const}; \tag{2}$$

$$\frac{d\varepsilon_x}{(\sigma_x - c\varepsilon_x) - (\sigma_y - c\varepsilon_y)} = \frac{d\varepsilon_y}{(\sigma_y - c\varepsilon_y) - (\sigma_x - c\varepsilon_x)} = \frac{d\varepsilon_{xy}}{2(\tau_{xy} - c\varepsilon_{xy})}; \tag{3}$$

$$\varepsilon_x = \frac{\partial u}{\partial x}, \qquad \varepsilon_y = \frac{\partial u}{\partial y}, \qquad \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{4}$$

where $\sigma_x, \sigma_y, \tau_{xy}$ are the stress components; $\varepsilon_x, \varepsilon_y, \varepsilon_{xy}$ are the strain components; u, v are the displacement components.

Fig. 1

At the initial instant of plastic flow, $\varepsilon_x = \varepsilon_y = \varepsilon_{xy} = 0$, the initial position of the yield condition can be represented as a circle (Fig. 1). During deformation,

Fig. 1

Figure 1: Fig. 1

the circle of the yield condition moves in the plane $\sigma_x - \sigma_y, 2\tau_{xy}$, its center coinciding with the point with coordinates $c(\varepsilon_x - \varepsilon_y), 2c\varepsilon_{xy}$.

Suppose that, at the initial instant of plastic flow at the given point of the body x, y, the stress state is interpreted by the vector OA shown in Fig. 1. If the coordinates of the point A are denoted by $\sigma_x^0 - \sigma_y^0, 2\tau_{xy}^0$, then the equation of the tangent to the circle of the yield condition passing through the point A can be written in the form

$$(\sigma_x^0 - \sigma_y^0)(\sigma_x - \sigma_y) + 4\tau_{xy}^0\tau_{xy} = 4k^2. \tag{5}$$

Suppose that the yield condition at each point of the body is interpreted by its own straight line (tangent to the yield circle), which shifts parallel to itself as the strain increases:

$$(\sigma_x^0 - \sigma_y^0)\left[(\sigma_x - c\varepsilon_x) - (\sigma_y - c\varepsilon_y)\right] + 4\tau_{xy}^0(\tau_{xy} - c\varepsilon_{xy}) = 4k^2. \tag{6}$$

Condition (6) is a linear approximation of the yield condition (2). We define the law of plastic flow by considering relation (6) as a plastic potential. We shall have

$$\frac{d\varepsilon_x}{\sigma_x^0 - \sigma_y^0} = \frac{d\varepsilon_y}{\sigma_y^0 - \sigma_x^0} = \frac{d\varepsilon_{xy}}{2\tau_{xy}^0}. \tag{7}$$

Taking into account that the quantities $\sigma_x^0, \sigma_y^0, \tau_{xy}^0$ retain a constant value at each point of the body, we integrate relations (7). Since at the initial instant of plastic flow $\varepsilon_x = \varepsilon_y = \varepsilon_{xy} = 0$, we finally obtain

$$\frac{\varepsilon_x}{\sigma_x^0 - \sigma_y^0} = \frac{\varepsilon_y}{\sigma_y^0 - \sigma_x^0} = \frac{\varepsilon_{xy}}{2\tau_{xy}^0}. \tag{8}$$

Let us note that the possibility of integrating relations (7) is also evident from the following considerations: the vector of the increment of plastic strains is orthogonal to parallel straight lines; consequently, at every point of the body the strains increase proportionally to one parameter.

Relations (8) determine the law of plastic deformation under the linearized plasticity condition (6).

Using the relations introduced, let us consider the problem of compression of a strip by rough plates.

Fig. 2

Figure 2: Fig. 2

Assume that a plastic layer of width $2h$ and length $2l$ is compressed by rigid rough plates (Fig. 2). It is known that

$$\begin{aligned} &\sigma_x^0 = -p - k\left(\xi - 2\sqrt{1-\eta^2}\right), \\ &\sigma_y^0 = -p - k\xi, \qquad \tau_{xy}^0 = k\eta \quad \left(\xi = \frac{x}{h},\ \eta = \frac{y}{h}\right), \end{aligned} \tag{9}$$

where p is a constant.

Fig. 2

Relations (8) coincide completely with the expressions of the associated flow law for an ideally plastic material under stresses $\sigma_x^0, \sigma_y^0, \tau_{xy}^0$ (with the only difference that, in the case under consideration, $\varepsilon_x, \varepsilon_y, \varepsilon_{xy}$ are strain components and not strain rates). Therefore, if the expressions for the displacements are taken to coincide with the expressions for the velocities in the ideally plastic flow of the strip, then the boundary conditions for the displacements and relations (8) will be completely satisfied.

Thus, take (7) as

$$u = Q + a\left(\xi - 2\sqrt{1-\eta^2}\right), \qquad v = -a\eta, \qquad Q = \text{const}, \tag{10}$$

where a is the displacement (settlement) of the plates along the y-axis.

From (10) we obtain

$$\varepsilon_x = \alpha, \qquad \varepsilon_y = -\alpha, \qquad \varepsilon_{xy} = \frac{\alpha\eta}{\sqrt{1-\eta^2}} \quad \left(\alpha = \frac{a}{h}\right). \tag{11}$$

Using expressions (9), (11), we rewrite condition (6) in the form

$$\sqrt{1-\eta^2}\,(\sigma_x - \sigma_y) + 2\eta\tau_{xy} = 2k + 2\alpha c\sqrt{1-\eta^2} - \frac{2\alpha c\eta^2}{\sqrt{1-\eta^2}}. \tag{12}$$

If we put $\tau_{xy} = k\eta$, then from the equilibrium equations (1) it follows that

$$\sigma_x = k\xi + \sigma_x^*(y), \qquad \sigma_y = \sigma_y(x). \tag{13}$$

Putting

Fig. 3

Figure 3: Fig. 3

$$\sigma_x = -P + \varphi(y), \qquad \sigma_y = -P - k\xi, \qquad P = \text{const}, \tag{14}$$

from (14), (13), (12) we find

$$\varphi(y) = 2k\sqrt{1-\eta^2} + \frac{2\alpha c}{1-\eta^2}. \tag{15}$$

The stress components take the form

$$\begin{aligned} \sigma_x &= -P - k\xi + 2k\sqrt{1-\eta^2} + \frac{2\alpha c}{1-\eta^2}, \\ \sigma_y &= -P - k\xi, \qquad \tau_{xy} = k\eta. \end{aligned} \tag{16}$$

Assuming the edge of the plate $x = 0$ to be free of stresses, we write the equilibrium condition for a part of the plate in the form

$$\int_0^{h-a} \sigma_x \, dy + \int_0^x (\tau_{xy})_{y=h} \, dx = 0, \tag{17}$$

whence

$$P = \frac{k\pi}{2} + \alpha c \ln\left(\frac{2-\alpha}{\alpha}\right). \tag{18}$$

It is obvious that $P = p = k\pi/2$ when $a = 0$. We also note that $Q = a(\pi/2 - l/h)$.

The pressure diagram on the layer from the plates is linear, with angular coefficient k/h, increasing as a function of the settlement of the plate according to formula (18).

Fig. 3

Figure 3 presents the dependence

$$q = \frac{1}{c}\left(P - \frac{k\pi}{2}\right)$$

on $\alpha = a/h$. The maximum is attained at $\alpha \approx 0.6$ and is equal to ≈ 0.5. It should be borne in mind that the deformations under consideration are small and the solution is valid for $\alpha^2 \ll 1$.

In the case where the plasticity condition is given in the form

$$(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2 = f\left[(\varepsilon_x - \varepsilon_y)^2 + 4\varepsilon_{xy}^2\right], \tag{19}$$

the corresponding linearized condition can be written in the form

$$(\sigma_x^0 - \sigma_y^0)(\sigma_x - \sigma_y^1) + 4\tau_{xy}^0\tau_{xy} = f\left[(\varepsilon_x - \varepsilon_y) + 4\varepsilon_{xy}^2\right]. \tag{20}$$

It is easy to see that the associated deformation law will have the form (8), and the problem under consideration can be solved under the adopted assumptions on the distribution of the tangential stresses.

Analogous results can be obtained in the case where the initial yield surface undergoes isotropic expansion and translation.

Voronezh State University

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Note: Figure translations are in progress. See original paper for figures.