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Abstract

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PHYSICS

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ON THE THEORY OF RADIATIVE PRODUCTION OF ELECTRON-POSITRON PAIRS ON A NUCLEUS

(Presented by Academician Ya. B. Zel' dovich on 10 VII 1962)

A theoretical analysis of the process of radiative pair production on nuclei: γ+nucleus $\rightarrow$ recoil nucleus$+e^{+}+e^{-}+\gamma'$ was carried out in several papers ([1−3]). Various special cases were considered in them. The purpose of the present article is to find a general expression for the differential cross section of the indicated process in the first nonvanishing approximation of perturbation theory (in order of magnitude it is equal to $\sim 1/137$ of the pair-production cross section).

Let $k_1(\omega_1, \mathbf{k}_1)$, $k_2(\omega_2, \mathbf{k}_2)$, $p_1(\varepsilon_1, \mathbf{p}_1)$, $p_2(\varepsilon_2, \mathbf{p}_2)$, $q(0, \mathbf{p}_1{+}\mathbf{p}_2{+}\mathbf{k}_2{-}\mathbf{k}_1)$ denote the energy-momentum vectors, respectively, of the incident and emitted photons, the positron, the electron, and the recoil nucleus (we neglect the kinetic energy of the recoil nucleus).

In the first nonvanishing approximation of perturbation theory, the contribution to the matrix element will be given by six diagrams, which are obtained by all possible permutations of the photon lines (with fixed electron and positron lines).

After summation over the spins of the final particles and averaging over the spins of the initial particles, the expression for the differential cross section of the process is written as follows*:

$$d\sigma = -r_0^2 \frac{Z^2\alpha^2|\mathbf{p}_1|\,|\mathbf{p}_2|\,|\mathbf{k}_2|\,d\Omega_{p_1}\,d\Omega_{p_2}\,d\Omega_{k_2}\,d\varepsilon_1\,d\varepsilon_2}{2(2\pi)^4\omega_1 q^4}\frac{1}{16}\,\mathrm{Sp}\,F. \tag{1}$$

Here

$$
\begin{aligned}
\operatorname{Sp} F = \operatorname{Sp} \Bigg\{ & \frac{\gamma_\nu(\hat{l}_1+1)\gamma_0(\hat{f}_1+1)\gamma_\mu}{-p_2k_2\cdot p_1k_1} + \frac{\gamma_\nu(\hat{l}_1+1)\gamma_\mu(\hat{f}_2+1)\gamma_0}{-p_2k_2[p_1q-q^2/2]} + \frac{\gamma_0(\hat{l}_2+1)\gamma_\nu(\hat{f}_1+1)\gamma_\mu}{p_1k_1[p_2q-q^2/2]} \\
& + \frac{\gamma_\mu(\hat{l}_3+1)\gamma_\nu(\hat{f}_2+1)\gamma_0}{p_2k_1[p_1q-q^2/2]} + \frac{\gamma_\mu(\hat{l}_3+1)\gamma_0(\hat{f}_3+1)\gamma_\nu}{-p_1k_2\cdot p_2k_1} + \frac{\gamma_0(\hat{l}_2+1)\gamma_\mu(\hat{f}_3+1)\gamma_\nu}{-p_1k_2[p_2q-q^2/2]} \Bigg\} (1-\hat{p}_1) \\
\times \Bigg\{ & \frac{\gamma_\mu(\hat{f}_1+1)\gamma_0(\hat{l}_1+1)\gamma_\nu}{-p_2k_2\cdot p_1k_1} + \frac{\gamma_0(\hat{f}_2+1)\gamma_\mu(\hat{l}_1+1)\gamma_\nu}{-p_2k_2[p_1q-q^2/2]} + \frac{\gamma_\mu(\hat{f}_1+1)\gamma_\nu(\hat{l}_2+1)\gamma_0}{p_1k_1[p_2q-q^2/2]} \\
& + \frac{\gamma_0(\hat{f}_2+1)\gamma_\nu(\hat{l}_3+1)\gamma_\mu}{p_2k_1[p_1q-q^2/2]} + \frac{\gamma_\nu(\hat{f}_3+1)\gamma_0(\hat{l}_3+1)\gamma_\mu}{-p_1k_2\cdot p_2k_1} + \frac{\gamma_\nu(\hat{f}_3+1)\gamma_\mu(\hat{l}_2+1)\gamma_0}{-p_1k_2[p_2q-q^2/2]} \Bigg\} (\hat{p}_2+1);
\end{aligned}
\tag{2}
$$

$l_1 = p_2 + k_2, \quad l_2 = p_2 - q, \quad l_3 = p_2 - k_1, \quad f_1 = -p_1 + k_1, \quad f_2 = -p_1 + q,$

$f_3 = -p_1 - k_2, \quad q = p_1 + p_2 + k_2 - k_1,$ the scalar product $ab = a_0b_0\left[1 - \frac{|\mathbf{a}|\,|\mathbf{b}|}{a_0b_0}\cos(ab)\right], \quad r_0 = \alpha = e^2$

* In what follows, Feynman notation is used throughout (see, for example, ([4])); energies and momenta are expressed in units of the electron rest mass.

Generally speaking, after multiplication in (2) it is necessary to calculate 21 traces*:

$$
\operatorname{Sp} F = \frac{\operatorname{Sp} 11}{\gamma_{11}} + \frac{\operatorname{Sp} 22}{\gamma_{22}} + \cdots + \frac{\operatorname{Sp} 66}{\gamma_{66}} + 2\left[\frac{\operatorname{Sp} 12}{\gamma_{12}} + \cdots + \frac{\operatorname{Sp} 23}{\gamma_{23}} + \cdots + \frac{\operatorname{Sp} 34}{\gamma_{34}} + \cdots + \frac{\operatorname{Sp} 45}{\gamma_{45}} + \cdots + \frac{\operatorname{Sp} 57}{\gamma_{56}}\right]. \tag{3}
$$

However, in fact one may restrict oneself to finding expressions only for 7 traces: 11, 12, 14, 15, 22, 23, 24, since from them the expressions for the remaining traces are obtained by simple substitutions (see Table 1). The explicit form of the denominators γ_{ik} is easily obtained from (2), for example $\gamma_{13} = -p_2k_2(p_1k_1)^2[p_2q - q^2/2]$, etc.

Table 1

Connection of traces	Form of substitution	Connection of traces	Form of substitution
$11 \to 55$	$l_1 \to l_3,\ f_1 \to f_3$	$15 \to 26$	$l_1 \to f_2,\ f_1 \to -p_1,\ -p_1 \to f_3 f_3 \to l_2,\ l_3 \to p_2,\ p_2 \to l_1$
$12 \to 13$	$l_1 \leftarrow f_1,\ -p_1 \leftarrow p_2,\ f_2 \to l_2$	$15 \to 34$	$l_1 \to p_2,\ f_1 \to l_2,\ -p_1 \to f_1 f_3 \to -p_1,\ l_3 \to f_2,\ p_2 \to l_3$
$12 \to 45$	$l_1 \to l_3,\ f_1 \to f_3$	$22 \to 33$	$l_1 \to f_1,\ f_2 \to l_2,\ -p_1 \rightleftarrows p_2$
$12 \to 56$	$f_1 \to l_3,\ l_1 \to f_3,\ -p_1 \leftarrow p_2,\ f_2 \to l_2$	$22 \to 44$	$l_1 \to l_3$
$14 \to 16$	$l_1 \rightleftarrows f_1,\ -p_1 \rightleftarrows p_2,\ f_2 \to l_2,\ l_3 \to f_3$	$22 \to 66$	$l_1 \to f_3,\ f_2 \to l_2,\ -p_1 \to p_2$
$14 \to 35$	$l_1 \to f_3,\ f_1 \rightleftarrows l_3,\ -p_1 \leftarrow p_2,\ f_2 \to l_2$	$23 \to 46$	$l_1 \to l_3,\ f_1 \to f_3$
$14 \to 25$	$l_1 \leftarrow l_3,\ f_1 \to f_3$		
$24 \to 36$	$l_1 \to f_3,\ f_2 \to l_2,\ l_3 \to f_1,\ -p_1 \rightleftarrows p_2$		

Calculations of the basic traces lead to the following result:

$$\begin{aligned}\frac{1}{16}\,\mathrm{Sp}\,11 = {} & 2p_2l_1\{\overline{p_1l_1}(f_1^2-1) - 2\overline{l_1f_1}(p_1f_1+2) - 2(p_1f_1+1)\} \\ & + l_1^2\{2p_1f_1(\overline{p_2f_1}+2) + \overline{p_1p_2}(1-f_1^2) + 4(\overline{p_2f_1}+f_1^2+1)\} \\ & - 4f_1^2(p_2l_1 + \overline{p_1l_1} - \tfrac{1}{4}\,\overline{p_1p_2} - 1) - 2p_1l_1(\overline{p_2l_1} - 4\overline{l_1f_1} - 2) \\ & + 4(4\overline{l_1f_1} - \overline{p_2f_1} + \overline{p_1l_1} - \tfrac{1}{4}\,\overline{p_1p_2} + 1);\end{aligned}$$

$$\begin{aligned}\frac{1}{16}\,\mathrm{Sp}\,12 = {} & l_1^2\{f_1f_2\cdot p_1p_2 + \overline{f_2p_1}(f_1p_2+2) - \overline{f_1p_1}\cdot\overline{f_2p_2} - \overline{f_1p_2} + 2t_1(p_{20}+2f_{10}) - 2\} \\ & + 2l_1p_2\{\overline{f_1l_1}(1-f_2p_1) - \overline{f_1f_2}\cdot p_1l_1 + \overline{f_1p_1}\cdot\overline{f_2l_1} - p_1f_2 - 2t_1t_2 + 1\} \\ & + \overline{p_1f_1}(\overline{f_2p_2} - 4\overline{f_2l_1}) + \overline{p_1f_2}(4\overline{f_1l_1} - \overline{f_1p_2} + 2) + f_1f_2(4p_1l_1 - p_1p_2) \\ & + 2\left\{\tfrac{1}{2}\,\overline{f_1p_2} - 2\overline{l_1f_1} + t_1(2t_2 + 2l_{10} - p_{20}) - 1\right\};\end{aligned}$$

$$\frac{1}{16}\,\mathrm{Sp}\,14 = \alpha_{14} + \beta_{14} + \overline{p_2 f_2}(l_3 l_1 + \overline{f_1 l_3} + 2l_{10}p_{10}) + \overline{p_2 l_3}(l_1 p_1 + \overline{f_1 p_1} - f_2 f_1 - \overline{f_2 l_1})$$
$$+ \overline{f_2 l_3}(\overline{p_2 l_1} + f_1 p_2) - \overline{p_1 p_2}(l_1 l_3 + \overline{f_1 l_3}) - p_1 l_3(f_1 p_2 + \overline{l_1 p_2}) - \overline{l_1 f_1}(\overline{f_2 p_1} - 1)$$
$$- l_1 p_1 \cdot \overline{f_1 f_2} + \overline{l_1 f_2} \cdot \overline{f_1 p_1} + 2l_{30}(p_1 p_2 f_{20} - p_1 f_2 p_{20})$$
$$+ 2p_{20}(l_1 f_1 l_{30} - l_1 l_3 f_{10} - f_1 l_3 l_{10} - l_{30})$$
$$+ (f_2 l_1 - \overline{p_1 f_2} - \overline{p_1 l_1} + 1)(p_2 f_1 + l_3 f_1)$$
$$+ (\overline{f_1 l_1} + \overline{p_1 l_1} + f_1 p_1 + 1)(\overline{f_2 p_2} + \overline{l_3 f_2})$$
$$+ (f_1 p_1 - \overline{l_1 f_2} - \overline{p_1 f_2} + 1)(\overline{p_2 l_1} + \overline{l_3 l_1})$$
$$- (f_2 l_1 + \overline{f_1 f_2} + \overline{f_1 l_1} + 1)(p_1 p_2 + l_3 p_1) - \overline{f_2 p_1} - 2t_1 t_2 + 1;$$

* In the notation ik, the first number corresponds to the ordinal number of the factor in the first curly bracket of expression (2), and the second number to that in the second curly bracket.

$$\frac{1}{16}\,\mathrm{Sp}\,15 = 2p_1 p_2(l_1 f_1 \cdot \overline{f_3 l_3} + l_1 f_3 \cdot \overline{f_1 l_3} - \overline{l_1 l_3} \cdot \overline{f_1 f_3}) + p_1 p_2(2\overline{l_1 f_1} - \overline{l_1 f_3} + 2) -$$
$$- f_1 p_1(l_1 f_3 + \overline{l_1 l_3}) + f_1 l_3(l_1 p_1 + \overline{l_1 f_3} + l_1 p_2 - p_1 f_3 - \overline{f_3 p_2} - 2) +$$
$$+ f_3 p_2(\overline{l_1 f_1} - l_1 p_1 - \overline{f_1 p_1} + 1) + \overline{f_1 f_3}\,(p_1 p_2 + l_1 l_3) +$$
$$+ \overline{f_1 f_3}\,(l_1 p_1 + l_1 p_2 + l_3 p_1 + l_3 p_2 - 2) - l_1 f_1(p_1 l_3 + \overline{p_1 f_3} + f_3 l_3 + l_3 p_2 - 1) +$$
$$+ f_1 p_2(\overline{l_1 l_3} - p_1 l_3 + \overline{l_3 f_3} - \overline{p_1 f_3} + 1) +$$
$$+ l_1 f_3(l_3 p_1 - \overline{f_1 p_2} + l_3 p_2 + 2l_{30} f_{10} - 2p_{10} p_{20} - 2) - 2l_1 f_1(f_{30} + p_{20}) l_{30} -$$
$$- \overline{l_1 l_3}\,(p_1 f_3 + \overline{f_3 p_2} + 2) + \overline{l_1 p_2}\,(f_3 p_1 + \overline{f_3 l_3} - p_1 l_3 + 1) + l_1 l_3(\overline{p_2 p_1} + 2p_{20} f_{10}) +$$
$$+ \overline{l_3 f_3}\,(2p_1 p_2 - \overline{f_1 p_1} - l_1 p_1 + 1) + \overline{f_1 l_3} \cdot p_1 p_2 - (\overline{f_1 p_1} - 1) p_2 l_3 +$$
$$+ 2f_{30} p_{10} l_1 p_2 - l_1 p_1 \cdot \overline{p_2 l_3} - \overline{f_3 p_1} - l_3 p_1 - l_1 p_1 - \overline{f_1 p_1} + 1;$$

$$\frac{1}{16}\,\mathrm{Sp}\,22 = l_1^2\left[2f_2 p_2(\overline{p_1 f_2} - 1) + f_2^2(4 - p_1 p_2) - 4(2\overline{f_2 p_1} - 1) + \overline{p_1 p_2}\right] +$$
$$+ 2l_1 p_2\left[\overline{p_1 l_1}(f_2^2 - 1) + 2p_1 f_2(2 - l_1 f_2) - 2(f_2^2 + 1 - l_1 f_2)\right] -$$
$$- f_2^2\left[4(\overline{l_1 p_1} - 1) - \overline{p_1 p_2}\right] - 2\overline{f_2 p_1}\left[f_2 p_2 + 4(1 - l_1 f_2)\right] - p_1 p_2 +$$
$$+ 2f_2 p_2 + 4(\overline{l_1 p_1} - 2l_1 f_2 + 1);$$

$$\frac{1}{16}\,\mathrm{Sp}\,23 = -\alpha_{23} + 2t_1\left[p_2 l_2 t_2 - (f_1 l_2 + l_1 l_2) p_{20} - (l_1 p_2 + f_1 p_2) t_{20} + 2t_3 - t_2\right] +$$
$$+ 4t_1 t_3 l_1 f_1 + 2t_3\left[(f_2 f_1 + l_1 f_2) p_{10} + (p_1 f_1 + l_1 p_1) f_{20} - p_1 f_2 t_2 - t_2\right] +$$
$$+ \overline{p_1 p_2} \cdot \overline{f_2 l_2} - p_1 l_2 \cdot f_2 p_2 - \overline{p_1 t_2}\,(\overline{p_2 l_2} + 1) - l_1 p_1 \cdot f_2 f_1 - \overline{l_1 t_1}\,(\overline{p_1 f_2} - 1) +$$
$$+ \overline{l_1 f_2} \cdot p_1 f_1 + p_2 l_2(1 + \overline{l_1 f_1}) + l_1 l_2 \cdot f_1 p_2 - \overline{l_1 p_2} \cdot \overline{f_1 l_2} + 1;$$

$$\frac{1}{16}\,\mathrm{Sp}\,24 = 2l_1l_3\left[2p_2f_2(\overline{p_1f_2}-1)-\overline{p_1p_2}(f_2^2-1)+\frac{1}{2}\right]+ \\ + f_2^2\left(l_1p_2+l_1l_3+l_3p_2+\overline{p_1p_2}+\overline{l_1p_1}+\overline{l_3p_1}-2\right)- \\ -2\overline{f_2p_1}\left(f_2l_3+f_2p_2+f_2l_1+l_3p_2+l_1l_3+l_1p_2-2\right)+ \\ +2(l_3f_2+f_2p_2+l_1f_2-1)-\overline{l_3p_1}+l_1p_2-\overline{p_1p_2}-\overline{l_1p_1}+l_3p_2,$$

where

$$t_1 = f_{20}-p_{10}, \qquad t_2 = l_{10}+f_{10}, \qquad t_3 = l_{20}+p_{20};$$

$$\alpha_{14} = l_1\tilde{f}_1\left[\tilde{p}_1p_2\cdot f_2l_3+\tilde{p}_2l_3\cdot\tilde{p}_1f_2-\tilde{p}_2f_2\cdot\tilde{p}_1l_3\right]+ \\ +l_1\tilde{p}_2\left[\tilde{f}_1f_2\cdot\tilde{p}_1l_3-\tilde{f}_1p_1\cdot f_2l_3-\tilde{f}_1l_3\cdot\tilde{p}_1f_2\right]+ \\ +l_1p_1[\tilde{f}_1\tilde{p}_2\cdot f_2l_3+\tilde{f}_1l_3\cdot\tilde{p}_2f_2-\tilde{f}_1f_2\cdot\tilde{p}_2l_3]+l_1f_2[\tilde{f}_1p_1\cdot\tilde{p}_2l_3-\tilde{f}_1p_2\cdot\tilde{p}_1l_3-\tilde{f}_1l_3\cdot\tilde{p}_2p_1]+ \\ +l_1l_3[\tilde{f}_1p_2\cdot p_1f_2+\tilde{f}_1f_2\cdot\tilde{p}_1p_2-\tilde{f}_1p_1\cdot\tilde{p}_2f_2],$$

β_{14} is obtained from α_{14} by the substitutions $\tilde{p}_2\to\tilde{f}_2$, $\tilde{p}_1\to p_1$, $f_2\to p_2$, and α_{23} by the substitutions $\tilde{f}_1\to p_1$, $\tilde{p}_2\to\tilde{f}_2$, $\tilde{p}_1\to\tilde{f}_1$, $f_2\to p_2$, $l_3\to l_2$, with $\widetilde{ab}=\tilde{a}b=\overline{ab}$, $\widetilde{\tilde{a}b}=ab$.

Let us emphasize that, when obtaining the expressions for the other traces from the basic ones, the corresponding substitutions indicated in Table 1 must also be extended to the quantities $f_{10}, f_{20}, f_{30}, l_{10}, l_{20}, l_{30}, p_{10}, p_{20}$; for example, in order to obtain 46 from 23, along with the substitution $l_1\to l_3$, $f_1\to f_3$, one must make the substitution $l_{10}\to l_{30}$ and $f_{10}\to f_{30}$ (in this case $f_{10}=-p_{10}+k_{10}$, etc.).

To compute the value of the differential cross section, one must find the trace quantities using the last expressions and Table 1, substitute the results obtained into (3), and the latter into (1).

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Note: Figure translations are in progress. See original paper for figures.