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Abstract

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PHYSICS

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ON THE DETERMINATION OF ISOBARIC STATES IN MODELS WITH A FIXED NUCLEON

(Presented by Academician N. N. Bogolyubov on 29 VIII 1962)

1. At the present time, in connection with the emergence of new ideas concerning the peculiarities in the behavior of amplitudes of various physical processes, interest has increased in the study of excited states of interacting systems. Since the description of relativistic particles interacting with one another is very complicated, the consideration of such simplified—even nonrelativistic—models is of particular interest, for which the calculation of the effect of interest to us can be carried out completely. We shall be interested in the isobaric states of a system containing an extended fixed nucleon strongly interacting with charged mesons (for simplicity, neutral mesons will not be considered). Then, as in papers ([1],[2]), the Hamiltonian of the system can be written in the form:

$$\mathcal{H} = \sum_{(k)} \omega_k \left(b_{k+}^+ b_{k+} + b_{k-}^+ b_{k-} \right) - g \left(Q\tau + \tau^+ Q^+ \right), \tag{1}$$

where

$$Q = \sum_{(k)} u_k \left(b_{k+} + b_{k-}^+ \right), \qquad Q^+ = \sum_{(k)} u_k \left(b_{k-} + b_{k+}^+ \right), \qquad u_k = \frac{\lambda_k}{\sqrt{2\omega_k}}, \tag{2}$$

τ, τ^+ are the creation and annihilation operators of the nucleon charge, possessing the property: $\tau\tau^+ + \tau^+\tau = 1$; b_{k+}^+ (b_{k+}), b_{k-}^+ (b_{k-}) are the creation (annihilation) operators of positive and negative mesons with momentum k; $\lambda_k = \lambda_k^* = \lambda(k^2)$ is the nucleon form factor and $g \gg 1$ is the coupling constant.

In contrast to paper ([2]), where a model was considered in which the degeneracy with respect to the possible charge states was removed by adding to the Hamiltonian (1) a term $\nu(Q-Q^{+})^2$, which does not conserve the total charge of the system, we shall study the system (1), degenerate with respect to charge, and determine the dependence of its energy on the charge. Since the transition to a point source in the degenerate case leads, upon renormalization of the coupling constant g, to a logarithmic divergence of the form $\frac{1}{g^2}\ln\frac{L}{\mu}$, where L is the cutoff momentum and μ is the meson mass, we shall consider only a smeared nucleon with form factor $\lambda_k \to 0$ as $k^2 \to \infty$.

In the present work we propose, as it seems to us, a simple and convenient method for determining the isobaric states of the system, not connected with the use of transformations of various types, on which the works ([1]) were based.

2. In the study of effects connected with the degeneracy of states of a system (arising, for example, as a consequence of invariance with respect to translations, rotations, etc.), it is necessary, as shown in paper ([3]),

consider, in general form, transformations of the dynamical variables that preserve the basic properties of the system. In this case it turns out that the arbitrariness allowed in these transformations, which is directly connected with the degeneracy, makes it possible to determine concrete physical characteristics of the system (for example, the effective mass of the electron in the polaron problem ([3]), etc.). However, this same nonuniqueness can be introduced directly into the Hamiltonian of the system in the form of a sum of products of Lagrange multipliers by integrals of motion ([4]). In this connection, let us write the Hamiltonian of the system in the form:

$$\mathcal{H} = \sum_{(k)} \omega_{k+} b^{+}_{k+} b_{k+} + \sum_{(k)} \omega_{k-} b^{+}_{k-} b_{k-} - g\left(Q\tau + \tau^{+}Q^{+}\right) + \Omega\left(q - \tau\tau^{+}\right), \qquad (3)$$

where q is the conserved total charge of the system, $\omega_{k\pm} = \omega_k \mp \Omega$, and the Lagrange multiplier Ω is determined from the equation

$$q = \sum_{(k)} \langle b^{+}_{k+} b_{k+}\rangle - \sum_{(k)} \langle b^{+}_{k-} b_{k-}\rangle + \langle \tau\tau^{+}\rangle. \qquad (4)$$

Here and everywhere below the averaging $\langle ...\rangle$ is taken over the ground state of the modified Hamiltonian (3).

Considering the Heisenberg equations of motion for the operators, one can obtain the following relation between the mean values of the boson and fermion operators:

$$\begin{aligned} \langle b_{k+}\rangle &= \frac{gu_k}{\omega_{k+}}\langle \tau^{+}\rangle, \quad \langle b_{k-}\rangle = \frac{gu_k}{\omega_{k-}}\langle \tau\rangle, \\ \langle b^{+}_{k+}\rangle &= \frac{gu_k}{\omega_{k+}}\langle \tau\rangle, \quad \langle b^{+}_{k-}\rangle = \frac{gu_k}{\omega_{k-}}\langle \tau^{+}\rangle. \end{aligned} \qquad (5)$$

Therefore, introducing new boson operators a_{k+} and a_{k-} and their conjugates:

$$\begin{aligned} b_{k+} &= \langle b_{k+}\rangle + a_{k+}, \quad b_{k-} = \langle b_{k-}\rangle + a_{k-}, \\ b^+_{k+} &= \langle b^+_{k+}\rangle + a^+_{k+}, \quad b^+_{k-} = \langle b^+_{k-}\rangle + a^+_{k-}, \end{aligned} \tag{6}$$

we shall have:

$$\langle a_{k+}\rangle = \langle a_{k-}\rangle = \langle a^+_{k+}\rangle = \langle a^+_{k-}\rangle = 0, \tag{7}$$

which proves convenient for further calculations.

Transforming the Hamiltonian (3) to the new variables, let us write it in the form:

$$\begin{aligned} \mathcal{H} &= \mathcal{H}_0 + \mathcal{H}_1, \\ \mathcal{H}_0 &= g^2 I(\Omega)\langle\tau\rangle\langle\tau^+\rangle - g^2 I(\Omega)\{\langle\tau^+\rangle\tau + \tau^+\langle\tau\rangle\} + \Omega(q - \tau\tau^+), \\ \mathcal{H}_1 &= -g\sum_{(k)} u_k\{(a_{k+} + a^+_{k-})(\tau - \langle\tau\rangle) + (\tau^+ - \langle\tau^+\rangle)(a_{k-} + a^+_{k+})\} \\ &\quad + \sum_{(k)} \omega_{k+} a^+_{k+} a_{k+} + \sum_{(k)} \omega_{k-} a^+_{k-} a_{k-}, \end{aligned} \tag{8}$$

where

$$I(\Omega) = \sum_{(k)} \frac{\lambda_k^2}{\omega_k^2 - \Omega^2}. \tag{9}$$

Here $\mathcal{H}_0$ is the principal part of the Hamiltonian, having the highest order g^2.

To determine the mean value of the system energy E_0 in the leading approximation, it is sufficient to take into account only $\mathcal{H}_0$:

$$E_0 = \langle\mathcal{H}_0\rangle = -g^2 I(\Omega)\langle\tau\rangle\langle\tau^+\rangle + \Omega(q - \langle\tau\tau^+\rangle). \tag{10}$$

Similarly, in the same approximation we obtain:

$$q = 2g^2\langle\tau\rangle\langle\tau^+\rangle F(\Omega)\Omega + \langle\tau\tau^+\rangle, \tag{11}$$

where

$$F(\Omega) = \sum_{(k)} \frac{\lambda_k^2}{(\omega_k^2 - \Omega^2)^2}. \tag{12}$$

3. The expressions obtained for E_0 and q contain the mean values of the fermion operators $\langle\tau\rangle$, $\langle\tau^+\rangle$, and $\langle\tau\tau^+\rangle$, which can be determined by perturbation theory in inverse powers of the coupling constant g. For example, up to terms of order $1/g^2$, one can obtain

$$\langle\tau\rangle = \langle\tau^+\rangle = \frac{1}{2}\left\{1 + \frac{1}{2g^2 I^2(0)}\, {}_{A_0}\langle(Q - Q^+)^2\rangle_{A_0}\right\}, \tag{13}$$

where the averaging ${}_{A_0}\langle\cdots\rangle_{A_0}$ is performed over the ground state A_0 of the Hamiltonian

$$\mathcal{H} = \sum_{(k)} \omega_k (a^+_{k+} a_{k+} + a^+_{k-} a_{k-}) + \frac{1}{4I(0)}(Q - Q^+)^2. \tag{14}$$

The mean value ${}_{A_0}\langle(Q - Q^+)^2\rangle_{A_0}$ can be determined by the method of u, v-transformations ([5]), or else by direct computation of the spectral density for the Green function:

$${}_{A_0}\langle\langle Q - Q^+ \mid Q - Q^+\rangle\rangle_{A_0}(E). \tag{15}$$

After simple calculations we obtain:

$$\langle\tau\rangle = \langle\tau^+\rangle = \frac{1}{2}\left\{1 - \frac{1}{\pi g^2}\int_\mu^\infty \frac{A(E)\,dE}{R^2(E) + A^2(E)}\right\}, \tag{16}$$

where

$$R(E) = \frac{E^2}{2\pi^2}\, P\int_0^\infty \frac{\lambda_k^2 k^2\, dk}{(k^2 + \mu^2)(E^2 - k^2 - \mu^2)}, \qquad A(E) = \frac{\lambda^2(E)}{4\pi}\sqrt{E^2 - \mu^2}. \tag{17}$$

The second term in formula (16) diverges logarithmically when the cutoff is removed; in this connection the expansion parameter of the theory should be taken as

$$\frac{1}{g^2}\ln\frac{L}{\mu}.$$

Analogously to (16), we obtain that, to the required order,

$$\langle\tau\tau^+\rangle = 1/2. \tag{18}$$

Introduce the renormalized coupling constant

$$g_r = g\langle\tau\rangle \tag{19}$$

and express through it the energy E_0 and the total charge of the system q:

$$E_0 = -g_r^2 I(\Omega) + \Omega(q - 1/2), \qquad q = 2g_r^2 F(\Omega)\Omega + 1/2. \tag{20}$$

Eliminating Ω from the system of equations (20), we obtain the dependence of the mean energy of the system E_0 on the charge q. If, when computing all convergent integrals, we set the form factor $\lambda_k = 1$, and in the calculation ···

If we retain it in the preceding terms, we obtain a result similar to the results of the works [1]:

$$E_0(q) = -g_r^2 I(0) + \frac{g_r^2 \mu}{4\pi}\left(\sqrt{1 - \left[\frac{4\pi(q-1/2)}{g_r^2}\right]^2} - 1\right). \tag{21}$$

This formula, apart from the first term, which depends only on the form factor λ_k and the coupling constant g_r and contains no other characteristics of the system, contains an isobaric term that depends on the charge of the system and retains its meaning also for a point nucleon. Expanding the expression for $E_0(q)$ in a series in powers of $1/g_r$, up to terms of order $1/g_r^2$, we obtain

$$E_0(q) = -g_r^2 I(0) + \frac{2\pi\mu}{g_r^2}\left(q - \frac{1}{2}\right)^2. \tag{22}$$

When the form factor $\lambda_k \neq 1$ is taken into account, this result is changed somewhat in the convergent integrals:

$$E_0(q) = -g_r^2 I(0) + \frac{(q-1/2)^2}{4g_r^2 F(0)}. \tag{23}$$

The structure of this formula corresponds to the expansion of each of the equations of the system (20) in powers of Ω/ω_k, up to terms of second order. The quantity standing in the denominator of the second term,

$$J = 2g_r^2 F(0) \tag{24}$$

defines the isotopic moment of inertia of the system, and the isobaric energy $\Delta E_q = E_0(q) - E_0(1/2)$ can be interpreted as the energy of rotation of the system in isotopic space about the z axis, with angular velocity Ω and moment of inertia J:

$$\Delta E_q = \frac{1}{2} J\Omega^2. \tag{25}$$

[1] R. Serber, S. Dancoff, *Phys. Rev.*, **63**, 143 (1943); A. Pais, R. Serber, *Phys. Rev.*, **105**, 1636 (1957); H. Nickle, R. Serber, *Phys. Rev.*, **119**, 449 (1960).

A similar method can also be applied to the solution of more complicated problems. For example, in the symmetric scalar model (three types of mesons), in addition to the total charge q, which, up to the term 1/2, is the third component of the vector of isotopic spin $\mathbf{I}$, the square of the total isotopic spin of the system is also conserved. Therefore the isobaric states will depend on q and $\mathbf{I}^2$, and to determine them one must add to the Hamiltonian of the system, with undetermined multipliers, the operators q and $\mathbf{I}^2$. Let us only note that the calculations in this case turn out to be very complicated, since $\mathbf{I}^2$ is a fourth-degree form in the boson operators.

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References

Note: Figure translations are in progress. See original paper for figures.