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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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A Cruciform Wing of Finite Span in a Compressible Flow

(Presented by Academician L. I. Sedov, 12 I 1963)

Let us consider a thin cruciform wing consisting of four panels of arbitrary geometric shape. Each of the two sides of the panels performs arbitrary harmonic oscillations with frequency ω in a supersonic flow. We solve the problem of determining the potential of the disturbed velocity $\Phi(x_1, y, z, t)$ in the linear formulation (^{1}). Φ satisfies the equation

$$(V_\infty^2 - a^2)\frac{\partial^2\Phi}{\partial x_1^2} - a^2\frac{\partial^2\Phi}{\partial y^2} - a^2\frac{\partial^2\Phi}{\partial z^2} + \frac{\partial^2\Phi}{\partial t^2} + 2V_\infty\frac{\partial^2\Phi}{\partial x_1\partial t} = 0.$$

We seek the reduced potential

$$\varphi(x, y, z) = \Phi(x, y, z, t)\cdot e^{-i(\omega t + k_1 x)},$$

satisfying the equation

$$\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial y^2} - \frac{\partial^2\varphi}{\partial z^2} + \lambda^2\varphi = 0, \tag{1}$$

where

$$\lambda = \frac{\omega}{a\sqrt{M^2-1}}, \qquad k_1 = -\frac{M\omega}{a(M^2-1)}, \qquad x = \frac{x_1}{\sqrt{M^2-1}}.$$

We solve the Cauchy problem and determine φ in the first octant $y > 0,\ z > 0$. For the remaining octants the problem is solved analogously. Suppose that on the coordinate surfaces, on the side of positive values of y and z, the normal derivatives $\partial\varphi/\partial\eta$ and $\partial\varphi/\partial\zeta$ are prescribed.

Fig. 1

From an arbitrary point M and from the symmetric points M_2, M_3, M_4, draw characteristic cones against the flow, which in the first octant cut off from the wing surface the areas S_1, S_2, S_1', S_2' (Fig. 1). Construct fictitious panels symmetric to the first and second panels of the wing. Then the boundary conditions

Fig. 2

Figure 2: Fig. 2

on S_1' and S_1'', as well as on S_2' and S_2'', will be identical. By Hadamard's method (2), we write Green' s formulas for the regions cut off in the first octant by the characteristic cones with vertices at the points M, M_2, M_3, M_4. Adding these formulas, we obtain the basic formula for the potential φ:

$$\varphi(x,y,z) = -\frac{1}{\pi}\left\{\iint_{S_1+S_1''}\frac{(\partial\varphi/\partial\zeta)\cos(\lambda r)}{r}\,d\sigma + \iint_{S_2+S_2''}\frac{(\partial\varphi/\partial\eta)\cos(\lambda r)}{r}\,d\sigma\right\}, \tag{2}$$

where

$$r = \sqrt{(x-\xi)^2-(y-\eta)^2-(z-\zeta)^2}.$$

Let us note that the integration is carried out not only over the wing surfaces, but also over the regions extending over the surfaces of the fictitious extensions.

Let each point of the horizontal extension oscillate according to the law $\zeta = \operatorname{Re} f_1(\xi,\eta)e^{i\omega t}$, where f_1 is complex. Omitting the sign Re, we have

$$\frac{\partial\Phi(\xi,\eta,0,t)}{\partial\zeta} = V_\infty\frac{\partial}{\partial\xi}\left(f_1e^{i\omega t}\right) + \frac{\partial}{\partial t}\left(f_1e^{i\omega t}\right);$$

$$\frac{\partial\varphi(\xi,\eta,0)}{\partial\zeta} = V_\infty\frac{\partial f_1}{\partial\xi}e^{-ik_1\xi} + i\omega f_1e^{-ik_1\xi}.$$

Analogously, on the second extension,

$$\frac{\partial\varphi(\xi0\zeta)}{\partial\eta} = V_\infty\frac{\partial f_2}{\partial\xi}e^{-ik_1\xi} + i\omega f_2e^{-ik_1\xi}.$$

The first terms of the formulas correspond to the normal velocity due to oscillations of the local angles of attack, and the second terms to that due to translational oscillations. In general the phases do not coincide, as in flutter.

Fig. 2

The determination of the tip effects from subsonic leading edges is reduced to the inversion of integral equations, as in (3). Let us write the expressions for φ by formula (2) for points located in all four octants. These formulas will contain the unknown velocities θ due to the tip effects. We equate the formulas written for adjacent octants on the coordinate planes in the zones of influence of the tip effects, where φ is continuous. These equalities will be the integral equations. Since equation (1) is linear, the influence of the velocities $\theta_1(\xi, \eta)$ from the region LJN (Fig. 2) on the potential at the point $M(x, y, z)$ can be represented as the sum of the influences $\theta_1(\xi, \eta) = \theta_{11} + \theta_{12}$ from the oscillations of the horizontal extensions and from the oscillations of the vertical extensions. The quantity θ_{11} was found in (^{3}). We shall find θ_{12} for the simplest case, when in the neighborhoods of the points $M_1(0, 0, z_1)$ and $M_0(0, 0, -z_1)$ small areas $\Delta\sigma$ with antisymmetric oscillations are prescribed. $f_2 = p_2 + \dfrac{\xi + \eta}{2} q_2$; p_2 and q_2 are complex constants. The remaining parts of the wing are undisturbed.

We seek the solution in the form of a series in powers of ω. Expanding the products under the integrals (2) in series in ω, applying the mean-value theorem and discarding

small quantities of higher order, we obtain

$$\begin{aligned}\varphi(x, y, z) = &-\frac{1}{2\pi} \sum_{n=0}^{\infty} \omega^n \left\{ \sum_{k=0}^{m} \frac{(-1)^k}{(2k)!\,(V_\infty^2 - a^2)^k} \times \right. \\ &\times \left[\int_{x_n}^{p} (x-\xi)^{k-1/2}\, d\xi \int_{z^2/\xi}^{y - z^2/(x-\xi)} \theta_{12}^{(n-2k)} \left(y - \frac{z^2}{x-\xi} - \eta \right)^{k-1/2} d\eta \right. \\ &\quad \left. + \int_{x_p}^{x_1} (x-\xi)^{k-1/2}\, d\xi \int_{\psi(\xi)}^{y - z^2/(x-\xi)} \theta_{12}^{(n-2k)} \left(y - \frac{z^2}{x-\xi} - \eta \right)^{k-1/2} d\eta \right] \\ &\quad \left. + \frac{(-1)^m\, 2A_2^{(n-2m)} \Delta\sigma}{(2m)!\,(V_\infty^2 - a^2)^m} \left\{ [xy - (z - z_1)^2]^{m-1/2} + [xy - (z + z_1)^2]^{m-1/2} \right\} \right\},\end{aligned} \tag{3}$$

where $m = n/2$ or $m = (n-1)/2$, $A_2^{(0)} = V_\infty q_2$, $A_2^{(1)} = i p_2$, $\theta_{12} = \sum_{n=0}^{\infty} \theta_{12}^{(n)} \omega^n$.

Taking into account, in setting up the integral equations, that ω may be arbitrary, we equate the coefficients of ω^n, and for the region JPN we have

$$\int_{z_1^2/y_2}^{x_2} \frac{d\xi}{\sqrt{x_2 - \xi}} \int_{z_1^2/\xi}^{y_2} \frac{\theta_{12}^{(n)}\, d\eta}{\sqrt{y_2 - \eta}} = -\frac{(-1)^m 4 A_2^{(n-2m)} \Delta\sigma\, (x_2 y_2 - z_1^2)^{m-1/2}}{(2m)!\,(V_\infty^2 - a^2)^m} -$$

$$-\sum_{k=1}^{m} \frac{(-1)^k}{(2k)!\,(V_\infty^2 - a^2)^k} \int_{z_1^2/y_2}^{x_2} (x_2 - \xi)^{k-1/2}\, d\xi \int_{z_1^2/\xi}^{y_2} \theta_{12}^{(n-2k)} (y_2 - \eta)^{k-1/2}\, d\eta. \tag{4}$$

We single out a strip of small thickness α, to which the integration is not extended. We invert equations (4) by Abel’ s formula. For $n = 0$,

$$\int_{z_1^2/(\xi-\alpha)}^{y_2} \frac{\theta_{12}^{(0)}(\xi,\eta)\,d\eta}{\sqrt{y_2-\eta}} = -\frac{4A_2^{(0)}\Delta\sigma\sqrt{\alpha}}{\pi\sqrt{y_2}\sqrt{\xi-(z_1^2/y_2)-\alpha(\xi-z_1^2/y_2)}}. \tag{5}$$

Integrating k times with respect to the parameter y_2, we obtain, as $\alpha \to 0$,

$$\int_{z_1^2/\xi}^{y_2} \theta_{12}^{(0)}(\xi,\eta)(y_2-\eta)^{k-1/2}\,d\eta = \frac{(-4)(2k-1)!!\,A_2^{(0)}\Delta\sigma z_1(\xi y_2-z_1^2)^{k-1}}{2^k(k-1)!\,\xi^{k+1/2}}. \tag{6}$$

Similarly, for $n=2$ and $n=4$, we obtain

$$\int_{z_1^2/\xi}^{y_2} \theta_{12}^{(2)}(\xi,\eta)(y_2-\eta)^{k-1/2}\,d\eta = \frac{(2k-1)!!}{2^k k!}\,\frac{A_2^{(0)}\Delta\sigma z_1}{(V_\infty^2-a^2)\xi^{k+1/2}}(\xi y_2-z_1^2)^k, \tag{7}$$

$$\int_{z_1^2/\xi}^{y_2} \frac{\theta_{12}^{(4)}(\xi,\eta)\,d\eta}{\sqrt{y_2-\eta}} = -\frac{A_2^{(0)}\Delta\sigma z_1(\xi y_2-z_1^2)}{8(V_\infty^2-a^2)^2\sqrt{\xi}}. \tag{8}$$

In the calculations we used the equality

$$\lim_{\alpha\to 0}\int_\alpha^\beta F(\xi)\,\frac{\sqrt{\alpha}}{\xi\sqrt{\xi-\alpha}}\,d\xi = \pi F(0),$$

where $F(\xi)$ is an arbitrary integrable function, continuous at the point $\xi = 0$.

The method of singling out the strip may be arbitrary, but it is important that it be the same both in solving the integral equations and in the limiting representation of φ. The solution of the integral equations for the velocities in the region LJP is carried out analogously.

Since the coordinates y_2 and y_3 are arbitrary, we set them equal to $y - \dfrac{z^2}{x-\xi}$ and substitute (5), (6), (7), (8), etc., instead of the inner integrals (3). We obtain

$$\Phi(x,y,z,t) = e^{\,i(\omega t+k_1\frac{x+y}{2})}\,\frac{(-1)}{2\pi}\,\frac{(V_\infty q_2\omega^2+ip_2\omega^3)\Delta\sigma}{(V_\infty^2-a^2)}\times$$

$$\times\left\{2z_1\left[\frac{\sqrt{x-x_n}}{\sqrt{x_n}}-\frac{\sqrt{x-x_l}}{\sqrt{x_l}}\right]-\sqrt{xy-(z-z_1)^2}-\sqrt{xy-(z+z_1)^2}\right\}+\cdots$$

All coefficients at even powers of ω are proportional to q, i.e., depend on oscillations of the wing angle of attack, while those at odd powers are proportional to p, i.e., depend on translational oscillations. If the element $\Delta\sigma$ is located on the cantilever l, then the potential takes the form

$$\Phi(x,y,z,t) = e^{i(\omega t + k_1 \frac{x+y}{2})} \frac{\Delta\sigma}{2\pi} \sqrt{\psi(0)}\sqrt{x - x_l} \times$$

$$\times \left\{ \frac{\omega^2 V_\infty q_1 + \omega^3 i p_1}{2(V_\infty^2 - a^2)} + \frac{\omega^4 V_\infty q_1 + \omega^5 i p_1}{2^5 (V_\infty^2 - a^2)^2} \left[y(x_l - 2x) + \frac{\psi(0)}{3}(x_l + 2x) \right.\right.$$

$$\left.\left. + 3z^2 - 2\int_0^{x_l} \psi(p)\,dp - \frac{z^2 x}{x - x_l} \right] + \cdots \right\}.$$

Solving the integral equations a second time, we obtain the velocity as a result of the end effect. By the method of induction it is shown that

$$\theta_{11}^{(n)}(\xi,\eta) = \frac{A_1^{(n-2m)} \Delta\sigma \sqrt{\psi(0)}}{(V_\infty^2 - a^2)^m} \frac{\xi^{m-1}}{\sqrt{\eta - \psi(\xi)}} \sum_{k=0}^{m-1} (\eta - \psi(\xi))^k \sum_{s=0}^{\infty} c_{ks} \xi^s.$$

For example,

$$\theta_{11}^{(2)} = \frac{V_\infty q_1 \Delta\sigma \sqrt{\psi(0)}}{4\pi(V_\infty^2 - a^2)} \frac{1}{\sqrt{\eta - \psi(\xi)}},$$

$$\theta_{11}^{(4)} = -\frac{V_\infty q_1 \Delta\sigma \sqrt{\psi(0)}}{2^5 (V_\infty^2 - a^2)^2 \pi} \frac{1}{\sqrt{\eta - \psi(\xi)}} \left(\xi\eta - \xi \frac{\psi(\xi) + \psi(0)}{2} + \int_0^{\xi} \psi(p)\,dp \right).$$

In the same way one can determine the potential φ of star-shaped wings with identical or different dihedral angles. These angles are equal to π/n', where $n' = 1, 2, \ldots$. The combination of one "dihedral" angle $\pi/1$ and two angles $\pi/2$ gives a T-shaped wing. The combination of two dihedral angles $\pi/4$ and three angles $\pi/2$ gives a symmetric arrangement of a cross-shaped wing at the end of an ordinary wing, etc. Suppose there is a channel-like region whose normal section is a rectangle or a rectangular isosceles triangle, and on whose surface $\partial\varphi/\partial n$ is prescribed. For a point M inside this region, one can analogously derive the basic formula for φ. The number of hyperbolas over which the integration in the basic formula is carried out depends on the number of reflections from the walls of the characteristic cone from the point M. If there is a complicated spatial configuration of thin wings with supersonic and subsonic edges and it can be divided by imaginary planes into separate channel-like regions and dihedral angles, then for each region one can write the basic formula and, in the case of subsonic edges, formulate integral equations by equating the values of the potential from adjacent regions in the zones of influence of the end effect.

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Note: Figure translations are in progress. See original paper for figures.

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