Corresponding Member of the USSR Academy of Sciences É. I. Grigolyuk, P. P. Chulkov

1963

SovietRxiv

Abstract

Full Text

Corresponding Member of the USSR Academy of Sciences É. I. Grigolyuk, P. P. Chulkov

SMALL DEFORMATIONS, STABILITY, AND VIBRATIONS OF ASYMMETRIC THREE-LAYER PLATES WITH A RIGID CORE

It is shown below that the calculation of a thin three-layer plate of asymmetric structure with a rigid core for strength, vibrations, and stability reduces to a single partial differential equation of the sixth order. It was assumed that the load-bearing layers have constant but different thicknesses and are made of different isotropic materials; the core material was regarded as transversely isotropic. For the load-bearing layers the Kirchhoff–Love hypotheses were adopted. It was assumed that the core is incompressible in the transverse direction and that the displacements of its points in the tangential directions are approximated with sufficient accuracy by linear functions of the transverse coordinate. The deformations were assumed to be elastic. As an illustration, the stability of a polygonal freely supported plate under uniform compression is investigated. It is noted that, in the study of cylindrical longitudinal-transverse bending of a plate, the method of initial parameters can readily be introduced. The results developed in the paper can, in principle, be generalized without difficulty to the case of orthotropic material of the load-bearing layers and the core.

1. We take as the initial plane the middle plane of the core and refer it to the Cartesian coordinate system ox_1x_2. We shall count the positive normal coordinate z upward, i.e., in the direction of the first load-bearing layer.

Let h_1, h_2, and $2c$ be, respectively, the thicknesses of the first and second load-bearing layers and of the core; $2a_i$ the absolute shift of the bonding planes; w and u_i the normal and tangential displacements of points of the initial plane. Then the tangential displacements of points of the package are written in the following form:

$$u_i^z = \begin{cases} u_i + a_i - (z-c)\,\partial w/\partial x_i, & c \le z \le c + h_1, \\ u_i + \dfrac{z}{c}\,a_i, & -c \le z \le c, \\ u_i - a_i - (z+c)\,\partial w/\partial x_i, & -c - h_2 \le z \le -c. \end{cases} \tag{1,1}$$

The linear strains and shear angles are equal:

for the load-bearing layers

$$e_{ij}^k = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \pm \frac{1}{2}\left(\frac{\partial a_i}{\partial x_j} + \frac{\partial a_j}{\partial x_i}\right) - (z \mp c)\,\frac{\partial^2 w}{\partial x_i \partial x_j} \qquad (i, j, k = 1, 2); \tag{1,2}$$

for the core

$$e_{ij}^3 = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \frac{z}{2c}\left(\frac{\partial a_i}{\partial x_j} + \frac{\partial a_j}{\partial x_i}\right);$$

$$e_{i3}^3 = a_i/c + \partial w/\partial x_i.$$

According to Hooke' s law, the stresses in the load-bearing layers and in the core will be

$$\sigma_{ij}^k = \frac{E_k}{1-\nu_k^2}\left[(1-\nu_k)e_{ij}^k + \delta_{ij}\nu_k e^{\,k}\right], \qquad e^{\,k} = e_{11}^k + e_{22}^k, \tag{1,3}$$

$$\sigma_{i3}^3 = Ge_{i3}^3, \qquad \delta_{ii} = 1, \quad \delta_{12} = 0 \qquad (i, j = 1, 2;\ k = 1, 2, 3).$$

Here E_k and ν_k $(k = 1, 2, 3)$ are, respectively, the moduli of elasticity of the first of the kind, and Poisson' s ratios of the materials of the load-bearing layers and the core; G is the shear modulus of the core material for the planes $x_1oz,\ x_2oz$. We introduce the stress resultants and moments as follows:

$$T_{ij} = \sum_{k=1}^{3} T_{ij}^k, \qquad M_{ij}^+ = \sum_{k=1}^{2} M_{ij}^k, \qquad H_{ij} = M_{ij}^3 + c(T_{ij}^1 - T_{ij}^2),$$

$$T_{ij}^1 = \int_c^{c+h_1} \sigma_{ij}^1\,dz, \qquad T_{ij}^2 = \int_{-c-h_2}^{-c} \sigma_{ij}^2\,dz, \qquad T_{ij}^3 = \int_{-c}^{c} \sigma_{ij}^3\,dz, \qquad Q_i^3 = \int_{-c}^{c} \sigma_{i3}^3\,dz. \tag{1.4}$$

$$M_{ij}^1 = \int_c^{c+h_1} \sigma_{ij}^1(z-c)\,dz, \qquad M_{ij}^2 = \int_{-c-h_2}^{-c} \sigma_{ij}^2(z+c)\,dz, \qquad M_{ij}^3 = \int_{-c}^{c} z\sigma_{ij}^3\,dz.$$

Composing the variation of the total potential energy with allowance for transverse shear of the core for a plate subjected to an external normal pressure q, we obtain the equilibrium equations of a small element

$$\frac{\partial T_{1i}}{\partial x_1} + \frac{\partial T_{2i}}{\partial x_2} = 0, \qquad \frac{\partial H_{1i}}{\partial x_1} + \frac{\partial H_{2i}}{\partial x_2} = Q_i^3 \quad (i = 1, 2); \tag{1.5}$$

$$\frac{\partial^2 M_{11}^+}{\partial x_1^2} + 2\frac{\partial^2 M_{12}^+}{\partial x_1 \partial x_2} + \frac{\partial^2 M_{22}^+}{\partial x_2^2} + \frac{\partial Q_1^3}{\partial x_1} + \frac{\partial Q_2^3}{\partial x_2} = q. \tag{1.6}$$

Equations (1.5) are satisfied by introducing the displacement function F by the formulas

$$w = (\eta_1 c^2 \Delta - \mu_1)F, \qquad \alpha_i = \frac{\partial}{\partial x_i}(\eta_2 c^2 \Delta + \mu_1)cF,$$

$$u_i = \frac{\partial}{\partial x_i}(\eta_3 c^2 \Delta - \mu_2)cF + C_{i0} + C_{i1}x_1 + C_{i2}x_2. \tag{1.7}$$

Equation (1.6) leads to a differential equation of sixth order with respect to the displacement function

$$Bc^2(\lambda_4 \eta_5 - \eta_6 c^2 \Delta)\Delta\Delta F = q. \tag{1.8}$$

Here

$$\Delta = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2, \qquad B = E_1 h_1/(1-\nu_1^2) + E_2 h_2/(1-\nu_2^2),$$

$$\eta_1 = 3\lambda_3^2 + 4(\lambda_3 + \lambda_1\lambda_2), \qquad \mu_1 = \lambda_4(1 + 3\lambda_3),$$

$$\eta_2 = 3\lambda_3(\lambda_1 t_1 + \lambda_2 t_2) + 2\lambda_1\lambda_2(t_1 + t_2), \qquad \mu_2 = \lambda_4(\lambda_1 - \lambda_2 + \lambda_1 t_1 - \lambda_2 t_2),$$

$$\eta_3 = \lambda_3(\lambda_1 t_1 - \lambda_2 t_2) + 2\lambda_1\lambda_2(t_1 - t_2), \qquad \eta_5 = \eta_1 + 2\eta_2 + \eta_4, \tag{1.9}$$

$$\eta_4 = (1/3 + 4\lambda_3)(\lambda_1 t_1^2 + \lambda_2 t_2^2) + \lambda_1\lambda_2(t_1 + t_2)^2,$$

$$\eta_6 = \begin{vmatrix} 1 + 3\lambda_3 & \lambda_1 - \lambda_2 & \lambda_1 t_1 - \lambda_2 t_2 \\ \lambda_1 - \lambda_2 & 1 + \lambda_3 & \lambda_1 t_1 + \lambda_2 t_2 \\ \lambda_1 t_1 - \lambda_2 t_2 & \lambda_1 t_1 + \lambda_2 t_2 & \frac{4}{3}(\lambda_1 t_1^2 + \lambda_2 t_2^2) \end{vmatrix},$$

$$t_1 = h_1/2c, \qquad t_2 = h_2/2c, \qquad \lambda_1 = E_1 h_1/B(1-\nu_1^2), \qquad \lambda_2 = E_2 h_2/B(1-\nu_2^2),$$

$3\lambda_3 = 2E_3c/B(1-\nu_3^2), \qquad \lambda_4 = 2Gc/B, \qquad C_{i0},\ C_{i1},\ C_{i2}$ are arbitrary constants.

In accordance with the adopted hypotheses, the tangential stresses σ_{i3} in the core do not depend on the transverse coordinate. A more realistic distribution of these stresses, not contradicting the system of equations (1.5)–(1.6), is obtained by expressing them in terms of σ_{ij}^k with the aid of the equilibrium equations of a three-dimensional body, using the conditions on the plate surfaces and the coupling conditions of the load-bearing layers and the core. It then becomes clear that equations (1.5) are necessary and sufficient for a continuous distribution of these stresses over the height of the package. We note that these assertions are also valid for a three-layer shell.

Equation (1.8) is suitable both for a rigid and for a soft filler; in the latter case one must set $\lambda_3 = 0$.

If the height of the filler is much greater than the height of each of the load-bearing layers, then the proper moments of the load-bearing layers may be neglected, and equation (1.8) takes the form

$$Bc^2(\eta_1 + \eta_2)\Delta\Delta F = q. \tag{1.10}$$

2. As an example, let us consider the stability of a polygonal plate, freely supported along its contour, under uniform compression by a force N. Using arbitrary constants C_{i1}, C_{i2}, we find that the neutral plane is located at the distance

$$z_0 = c\,\frac{(\lambda_1 + \lambda_1 t_1)(1+\nu_1) - (\lambda_2 + \lambda_2 t_2)(1+\nu_2)}{3\lambda_3(1+\nu_3) + \lambda_1(1+\nu_1) + \lambda_2(1+\nu_2)} \tag{2.1}$$

from the initial plane. The stability equation is obtained by introducing into equation (1.8), instead of q, the reduced transverse pressure

$$N_1\frac{\partial^2 w}{\partial x_1^2} + 2N_{12}\frac{\partial^2 w}{\partial x_1 \partial x_2} + N_2\frac{\partial^2 w}{\partial x_2^2},$$

where N_1, N_2, N_{12} are the normal and shear forces in the neutral plane of the plate.

Then the stability equation is written in the form

$$Bc^2(\lambda_4\eta_5 - \eta_6 c^2\Delta)\Delta\Delta F - N(\eta_1 c^2\Delta - \mu_1)\Delta F = 0 \tag{2.2}$$

with boundary conditions $F = 0,\ \Delta F = 0,\ \Delta\Delta F = 0$ on the contour.

Using the invariant character of equation (2.2) and the boundary conditions, we obtain

$$N_{\rm cr} = Bc^2k^2\frac{\lambda_4\eta_5 + k^2c^2\eta_6}{\mu_1 + k^2c^2\eta_1},$$

where k^2 is determined from the equation

$$\Delta F + k^2F = 0$$

with the boundary condition $F = 0$ on the contour.

Thus, for example, for a rectangular plate with sides l_1 and l_2, and for a plate in the form of an equilateral triangle with side a, we have, respectively,

$$k^2 = m^2\pi^2/l_1^2 + n^2\pi^2/l_2^2 \qquad (m, n = 1, 2, ...),$$
$$k^2 = 16n^2\pi^2/3a^2 \qquad (n = 1, 2, ...).$$

The equation of the free vibrations of the plate has the form

$$Bc^2(\lambda_4\eta_5 - \eta_6c^2\Delta)\Delta\Delta F = \Omega(\eta_1c^2\Delta - \mu_1)\frac{\partial^2 F}{\partial t^2}.$$

Here

$$\Omega = \rho_1h_1 + \rho_2h_2 + 2\rho_3c;$$

ρ_1, ρ_2, and ρ_3 are the specific densities of the materials, respectively, of the first and second load-bearing layers and of the filler.

The natural frequencies of polygonal plates with a freely supported contour are expressed through k^2 in the following form:

$$\omega^2 = \frac{Bc^2}{\Omega}\,k^4\frac{\lambda_4\eta_5 + \eta_6c^2k^2}{\mu_1 + \eta_1c^2k^2}.$$

3. Let us note that, on the basis of equation (2.2), in the investigation of the longitudinal-transverse cylindrical bending of plates the method of initial parameters can easily be introduced.

Institute of Hydrodynamics
Siberian Branch of the Academy of Sciences of the USSR

Received
6 XII 1962

REFERENCES

1. A. Ya. **Aleksandrov**, A. E. **Bryukker**, M. M. **Kurshin**, A. P. **Prusakov**, *Calculation of Three-Layer Panels*, Moscow, 1960.

Note: Figure translations are in progress. See original paper for figures.