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Abstract

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GEOPHYSICS

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PLASTIC DEFORMATIONS IN A GRAVITATING SPHERE

According to geophysical data, seismic waves propagating in the Earth undergo reflection and refraction at the boundaries of certain layers. It is assumed that the aforementioned boundaries may be of two types: between chemically different substances and between different phases of one and the same substance. In particular, it is accepted ([1]) that the Earth' s core has a solid inner core G and an outer part of the core (layer E), in which transverse waves are not observed to propagate; this part is considered to be in a liquid molten state.

We wish to draw attention to yet another possible cause of the occurrence of a boundary separating layers: plastic failure of a deformed substance.

Let us first consider the classical problem of the distribution of stresses inside a stationary gravitating elastic sphere of constant density ([2]). From symmetry considerations, $\delta_{\theta\theta} = \delta_{\varphi\varphi}$, and for the stress tensor one may write

$$\sigma_{ik} = -p\delta_{ik} + \tau\left(n_i n_k - \frac{\delta_{ik}}{3}\right). \quad (1)$$

Here $p(r)$ is the pressure, and the function $\tau(r)$ characterizes the intensity of the shear stresses; $\mathbf{n} = \mathbf{r}/r$ is the unit vector of the radius vector. Assuming, for simplicity, that Hooke' s law is applicable everywhere, one can use it to express the stress tensor in terms of the deformation vector and its derivatives and, after solving the equilibrium equations, obtain formulas for the distribution of the intensity of shear stresses and of the pressure:

$$\tau = \frac{4\pi}{15}\frac{1-2\sigma}{1-\sigma}\gamma\rho r^2; \quad (2)$$

$$p = p_1 - \frac{2\pi}{9}\frac{1+\sigma}{1-\sigma}\gamma\rho r^2. \quad (3)$$

Here γ is the constant of the law of universal gravitation; σ is the Poisson coefficient of the material of the sphere; $p_1 = \frac{2\pi}{15}\frac{3-\sigma}{1-\sigma}\gamma\pi R_1^2$ is the pressure at the center of the sphere, determined from the condition

$$\sigma_{rr} = -p + {}^2/_3\tau = 0 \tag{4}$$

at the boundary of the sphere when $r = R_1$.

It is known that the intensity of shear stresses, characterized here by the function τ, cannot exceed a certain pressure-dependent limit $\tau^*(p)$. Generally speaking, the criterion should have the form $\Phi(\sigma_\varphi \sigma_\theta \sigma_r) = \Phi_{\text{cr}}$, but in our case $\sigma_\varphi = \sigma_\theta$, and it can be rewritten as was done above. It is essential that the values of τ^*, as a rule, increase with increasing pressure; according to (2) and (3), however, with increasing distance from the center of the sphere the pressure decreases, while the intensity of the shear stresses increases. Therefore, in sufficiently large bodies of any substance, on the outer side of the sphere there must arise a region of plastic deformation. Under shear stresses exceeding the critical value, the substance flows plastically until τ assumes the value $\tau^*(p)$ given by the criterion. In mechanical equilibrium the relation between p and τ is established not through the deformation vector according to Hooke' s law, but directly from the equation

$\tau = \tau^*(p)$, expressing the condition for the onset of plastic deformations. In this region, in the general equilibrium equation

$$\frac{\partial \sigma_{ik}}{\partial x_k} = \rho g_i$$

(g_i is the intensity of the gravitational field) one should substitute δ_{ik} from (1), while setting $\tau = \tau^*(p)$. This gives the equation

$$-\frac{dp}{dr}\left(1 - \frac{2}{3}\frac{d\tau^*}{dp}\right) + \frac{2\tau^*(p)}{r} = \rho g. \tag{5}$$

It is easy to obtain the solution for the case $\tau^* = \text{const}$, $\rho = \text{const}$:

$$p = 2\tau^* \ln r - \frac{2\pi}{3}\gamma\rho r^2 + C, \tag{6}$$

where the integration constant C is determined from the boundary condition $\sigma_{rr} = 0$ at $r = R_2$.

The solution in the inner, elastically deformed region is given in this case by formulas (2), (3), with the position of the R_1-boundary between the two regions determined by formula (2) from $\tau_{R_1} = \tau^*$. The constant p_1 is found from the condition of continuity of σ_{rr} and p at $r = R_1$. The equations usually used in geophysics are obtained from (5) if one sets $\tau^* = 0$, i.e., assumes that in the solid bodies composing the Earth, over the time of its existence, all shear stresses have completely relaxed. Unfortunately, we are not at present able to settle this fundamental question exhaustively; to do so it is necessary to study the dependence of τ^* on pressure, temperature, and loading time.

It seems possible that the outer region of the Earth' s core is not liquid, but plastically crushed; shear elastic waves in it must, evidently, propagate with a much larger attenuation coefficient than longitudinal waves. The boundary between the elastically deformed region and the plastically crushed region must be rather sharp, even if the material composing the core has a complex chemical composition. In principle, plastically crushed regions may occur in a layer of any composition at various depths. It may be assumed that the Earth' s mantle is also plastically crushed. The layers B, C, D observed in it are due to phase transformations in the material composing it. The behavior of plastically crushed material in very slow processes should not differ qualitatively from the behavior of a liquid (the term $2\tau^* \ln r$ in equation (6), when $\tau \ll p$, is a small, slowly varying correction). For rapid processes (seismic waves), plastically crushed material should behave as an elastic one. Let us emphasize once again that all the considerations developed are based on the assumption that, in the Earth over the time of its existence, shear stresses have not had time to decrease to vanishingly small values.

Let us note the important role that plastically crushed regions would have to play in the heat balance of the terrestrial sphere.

As is known, the depth $\sqrt{\chi t}$ of penetration of thermal disturbances, if the age of the Earth is taken as $t \sim 5 \cdot 10^9$ years and the thermal diffusivity as $\chi \sim 10^{-2}$, is of the order of hundreds of kilometers, i.e., $\sqrt{\chi t} \ll R$, where R is the radius of the Earth. Therefore, during the existence of the Earth, the nonequilibrium temperature distribution could not have become equalized, and this process must be continuing at the present time. When temperature is equalized and a nonuniformly heated elastically deformed sphere or liquid "drop" cools, there occurs, determined by the coefficient of thermal expansion, a change in dimensions and a conversion of gravitational energy into energy of elastic deformation. At the same time,

only a very small part is converted into heat, corresponding to viscous dissipation processes, quadratic in the small rates of deformation.

In the presence of plastically crushed regions the picture changes substantially. It is obvious that, under plastic deformation, energy of the same order as the change in elastic deformation is converted into heat; therefore the regions under consideration must behave as effective heat sources, in which a significant part of the change in the gravitational energy of the entire body is released. This effect is of first order of smallness with respect to the rates of deformation.

The gravitational energy per unit mass is enormous (of the order of gR). Its relative change due to compression during cooling is of the order of $\alpha \Delta T$ (α is the coefficient of thermal expansion) and is still very large. A consistent account of the reverse effect of the released heat on the deformations requires the joint solution of the equations of equilibrium and heat transfer.

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Note: Figure translations are in progress. See original paper for figures.