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Abstract

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RADIATION OF ELECTROMAGNETIC WAVES FROM A SYSTEM OF SEMI-INFINITE PLATES

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The problem of radiation from a system of semi-infinite thin plates has been considered by a number of authors; however, for the practically important case in which only one of the plane waveguides is excited, no solution had been given. In the present article the field in the far zone is found, as well as the influence of the excited waveguide on neighboring ones.

Let us consider the problem of radiation of electromagnetic waves from a system of semi-infinite thin metallic plates (we shall take the distance between plates to be equal to unity)

$$y = m - \frac{1}{2}, \quad x < 0, \quad m = \dots, -1, 0, 1, 2, \dots$$

We shall restrict ourselves to radiation of magnetic waves of the fundamental type. For this purpose we shall assume that

$$\varphi(x,y) \equiv E_z, \qquad H_x = -\frac{i}{k}\frac{\partial\varphi}{\partial y}, \qquad H_y = \frac{i}{k}\frac{\partial\varphi}{\partial x},$$

$$(\partial^2/\partial x^2 + \partial^2/\partial y^2 + k^2)\varphi = 0,$$

$$\varphi = 0$$

on the plates, and from each waveguide there is incident a wave

$$\varphi_n = c_n \cos\pi(y-n)e^{i\hat{k}x},$$

where

$$\hat{k} = \sqrt{k^2-\pi^2}, \quad k = 2\pi/\lambda, \quad n - \frac{1}{2} < y < n + \frac{1}{2}.$$

We divide all space into regions

$$n-\frac{1}{2}<y<n+\frac{1}{2}, \qquad -\infty<x<\infty,$$

and represent the field $\tilde{\varphi}_n$ in each region as $\tilde{\varphi}_n=\varphi_n+\varphi$.

We use the periodicity of the structure in y, i.e., we shall seek solutions satisfying the condition

$$\varphi_q(x,y+1)=e^{iq}\varphi_q(x,y).$$

In this case it is natural to choose the amplitudes of the incident waves as

$$c_n=\frac{1}{2\pi}e^{iq}.$$

We shall now use the method developed by Jones ([1]). Here we shall not carry out all the calculations, but shall give only the final result, since because of the periodicity condition the "matching" of the solutions need be performed only on one plate and its continuation, and therefore the course of the solution is analogous, for example, to the problem of radiation from a pair of parallel half-planes.

Using the notation (1), we obtain:

$$\varphi(x,y)=\frac{1}{4\pi i}\int_{-\infty}^{\infty}d\alpha\, e^{-i\alpha x}\left[e^{i\lambda(y-n)}e^{iqn}\frac{e^{-i\lambda/2}(e^{i\lambda}-e^{-iq})\Phi_+(\alpha)}{\sin\lambda}-e^{-i\lambda(y-n)}e^{iqn}\frac{e^{i\lambda/2}(e^{-i\lambda}-e^{-iq})\Phi_+(\alpha)}{\sin\lambda}\right], \tag{1}$$

$$n-\frac{1}{2}<y<n+\frac{1}{2}, \qquad -\infty<x<\infty,$$

where

$$\Phi_+(a)=\frac{i(1+e^{iq})}{4(a+\tilde{k})K_-(q,-\tilde{k})K_+(q,a)}, \qquad \lambda=\sqrt{k^2-a^2},$$

$$K_\pm(q,a)=(1-\cos q)^{1/2}e^{\pm i\frac{a}{\pi}\ln 2}\prod\frac{\left|(1-k^2/\lambda_p^2)^{1/2}\mp ia/\lambda_p\right|\left|(1-k^2/\gamma_p^2)^{1/2}\mp ia/\gamma_p\right|}{\left|(1-k^2/(\pi p)^2)^{1/2}\mp ia/\pi p\right|},$$

$$K_+(q,a)K_-(q,a)=\frac{(\cos\lambda-\cos q)\lambda}{\sin\lambda}, \qquad \lambda_p=2\pi(p-1)+q, \qquad \gamma_p=2\pi p-q.$$

The functions $K_+(a)$ and $K_-(a)$ are analytic, respectively, in the upper and lower half-planes, and $|K_+(a)| \sim |a|^{1/2}$ if $|a| \to \infty,\ 0 \le \arg a \le \pi$.

An arbitrary amplitude of the incident wave may be written as

$$c_n = \int_{-\pi}^{\pi} c_n(q)\,\gamma_q\,dq.$$

If we now require that only one waveguide with $n = 0$ be excited, i.e. $c_n = \delta_{0n}$, then $\gamma_q = 1$. The general solution, which is a superposition of particular solutions with weight γ_q, in this case will be

$$\varphi(x,y) = \int_{-\pi}^{\pi} \varphi_q(x,y)\,dq. \tag{2}$$

Substituting (2) into (1) and carrying out some transformations, we obtain

$$\varphi(x,y) = \frac{1}{8\pi}\int_{-\infty}^{\infty} d\alpha\, e^{-i\alpha x}\left[\frac{e^{i\lambda(y-n)}}{(a+\tilde{k})\sin\lambda}\int_0^{\pi} dq\,\frac{f_n(q,a)}{K_-(q,-\tilde{k})K_+(q,a)}\right.$$

$$\left.+\frac{e^{-i\lambda(y-n)}}{(a+\tilde{k})\sin\lambda}\int_0^{\pi} dq\,\frac{f_{-n}(q,a)}{K_-(q,-\tilde{k})K_+(q,a)}\right], \tag{3}$$

$$f_n(a,q) = e^{i\lambda/2}[\cos qn + \cos q(n+1)] - e^{-i\lambda/2}[\cos qn + \cos q(n-1)],$$

$$n - \tfrac{1}{2} < y \le n + \tfrac{1}{2}.$$

If $x < 0$, then, closing the path of integration in a in the upper half-plane, we obtain

$$\varphi(x,y) = \sum_{m=1}^{\infty} R_{nm}\cos\pi(2m-1)(y-n)e^{-ia_{2m-1}x} +$$

$$+\sum_{m=1}^{\infty}\widetilde{R}_{nm}\sin 2\pi m(y-n)e^{-ia_{2m}x}, \qquad n-\tfrac{1}{2} < y < n+\tfrac{1}{2}, \tag{4}$$

$$R_{n,m} = \frac{(-1)^m\pi(2m-1)}{a_{2m-1}(a_{2m-1}+\tilde{k})}\int_0^{\pi} dq\,\frac{\cos qn(1+\cos q)}{K_-(q,-\tilde{k})K_+(q,a_{2m-1})},$$

$$\widetilde{R}_{nm} = \frac{(-1)^{m+1}2\pi m}{a_{2m}(a_{2m}+\tilde{k})}\int_0^{\pi} dq\,\frac{\sin qn\sin q}{K_-(q,-\tilde{k})K_+(q,a_{2m})}, \tag{5}$$

Fig. 1

Figure 1: Fig. 1

$$a_m = \sqrt{k^2 - (\pi m)^2}.$$

R_{nm}, $\widetilde{R}_{nm}$ may be called the transformation coefficients of the fundamental wave incident from the waveguide with $n = 0$ into the m-th wave propagating in the n-th waveguide. The decrease of R_{nm} and $\widetilde{R}_{nm}$ with increasing number n is ensured by the oscillating factor $\sin qn$ or $\cos qn$ under the integral. For an estimate

For large n, let us write R_{nm} and $\widetilde{R}_{nm}$ in the form

$$R_{nm} = \int_0^\pi \cos qn \, \psi_n(q) \, dq, \qquad \widetilde{R}_{nm} = \int_0^\pi \sin qn \, \varphi_m(q) \, dq,$$

where $\psi_m(q)$ and $\varphi_m(q)$ have derivatives with respect to $q \sim 1$. Integrating twice by parts and taking into account that $\varphi_m(0) = \varphi_m(\pi) = 0$, we obtain

$$R_{nm} = [(-1)^n \psi'_m(\pi) - \psi'_m(0)] \frac{1}{n^2} + O\left(\frac{1}{n^4}\right), \tag{6}$$

$$\widetilde{R}_{nm} = [(-1)^n \varphi''_m(\pi) - \varphi''_m(0)] \frac{1}{n^3} + O\left(\frac{1}{n^5}\right). \tag{7}$$

Thus, R_{nm} decreases as $\frac{1}{n^2}$, while $\widetilde{R}_{nm}$ decreases as $\frac{1}{n^3}$.

Fig. 1

Formulas (6), (7) can already be used for $n = 3$; in this case the error will be ~ 0.01.

For $x > 0$ the field can be calculated in the far zone. Introducing the variables $x = r\cos\vartheta$, $y = r\sin\vartheta$, and also $\alpha = -k\cos\beta$ ($\beta = \mu + i\nu$), we write integral (3) in the form

$$\varphi(r, \vartheta) = \int_C d\beta \, \sin\beta \, e^{ikr\cos(\beta - \vartheta)} \times$$

$$\times \left[\psi_n(-k\cos\beta) + \psi_{-n}(-k\cos\beta)\right], \tag{8}$$

$$\psi_n(\alpha) =$$

$$= \frac{ke^{-i\lambda n}}{8\pi(\alpha+\tilde{k})\sin\lambda}\int_0^{\pi}\frac{f_n(\alpha,q)}{K_-(q,-\tilde{k})\,K_+(q,\alpha)}\,dq.$$

For real q, $\psi_n(\alpha)$ has cuts, as shown in Fig. 1 by dashed lines. However, considering q complex, one may change their position by changing the contour of integration in q (see Fig. 1).

Deforming, for large r, the contour C to the path of steepest descent, we obtain

$$\varphi(r,\vartheta) = \pi k\,[\psi_n(-k\cos\vartheta)+\psi_{-n}(-k\cos\vartheta)]\sin\vartheta\,H_0(kr)+I_n(\vartheta,r),$$

$$n-\tfrac{1}{2}<y<n+\tfrac{1}{2},\qquad n=\ldots,-1,0,1,\ldots,$$

where I_n is the integral along the cut (see Fig. 1).

Let us now calculate $\psi_n+\psi_{-n}$. Introducing the variable x by the formula $q=\sqrt{k^2-x^2}$, we note that the integrand will contain the factor $1/(x-k\cos\vartheta)$. Using the formula $1/(x-k\cos\vartheta-i\delta)=P[1/(x-k\cos\vartheta+i\pi\delta(x-k\cos\vartheta))]$, we note that as $r\to\infty$ one may always regard n as very large; therefore the integral in the sense of the principal value may be neglected, since, owing to the strongly oscillating factor, it will be of order $1/n$. Thus, one may integrate when $k>k\cos\vartheta>\tilde{k}$. If $\tilde{k}>k\cos\vartheta>0$, then one must introduce the variable x by the formula $q=-\sqrt{k^2-x^2}+2\pi$. The rest is analogous. Carrying out the integration, we obtain

$$\varphi(r,\vartheta)=\frac{\pi k^2\sin\vartheta\cos\vartheta\cos(k\sin\vartheta/2)}{2(\tilde{k}-k\cos\vartheta)\,K_-(\beta,-\tilde{k})\,K_+^1(\beta,-k\cos\vartheta)}.$$

whereas for $k>k\cos\vartheta>\tilde{k}$

$$\beta=k\sin\vartheta,\qquad K_+^1(\beta,\alpha)=\frac{K_+(\beta,\alpha)}{i\left[(1-k^2/\beta^2)^{1/2}-i\alpha/\beta\right]},$$

and for $\tilde{k}>k\cos\vartheta>0$

$$\beta=2\pi-k\sin\vartheta,\qquad K_+^1(\beta,\alpha)=\frac{K_+(\beta,\alpha)}{i\left[(1-k^2/(2\pi-\beta)^2)^{1/2}-i\alpha/(2\pi-\beta)\right]}.$$

We have neglected the integral along the cut, since for large r we have

$$|I|\sim 1/r^2.$$

Let us note that the final result has ceased to depend on n, and therefore it is valid for arbitrary ϑ. Physically this is obvious, since as $r \to \infty$ only a diverging wave should exist.

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.