CYBERNETICS AND CONTROL THEORY

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Abstract

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ON A CLASS OF TURING MACHINES

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In the present note a class of Turing machines is considered, related to machines without erasure, studied in ([1,2]). As a characteristic of the computational capabilities of such machines one takes the set of operators realized by them (see item 1°). These operators are compared with operators studied in automata theory and in related areas of algorithm theory (automaton operators ([3]), finite operators ([4])).

1°. **Definition 1.** An A**-machine** is a Turing machine (with a one-sided infinite tape) for which the following conditions are satisfied: a) its external alphabet is divided into two classes in such a way that, in accordance with the functional scheme of the machine, each letter of the first class may change either into itself or into one of the letters of the second class; each letter of the second class is changed only into itself; b) suppose that the tape is filled with an arbitrary sequence of letters of the first class; then each letter, after a finite number of cycles of operation of the machine, will be replaced by a letter of the second class. The letters of the first class will be called **input** letters, and the letters of the second class **output** letters.

Each A-machine realizes an effective operator which, in accordance with condition b), transforms all possible sequences $x(1), x(2), \dots, x(n), \dots$ of input letters into sequences $z(1), z(2), \dots, z(n), \dots$ of output letters. Here $x(n)$ (respectively, $z(n)$) denotes the input (output) letter written in the n-th cell of the tape (it is assumed that each cell is assigned a number giving its distance from the beginning of the tape). Whereas condition a) is trivially recognizable from the functional scheme of the machine, condition b) is not; specifically, the following theorem is true:

Theorem 1. *The set of all A-machines is not recursively enumerable.*

The proof of this fact is based on a result of Hao Wang ([1]).

2°. The study of operators realized by A-machines is conveniently carried out in terms of memory and anticipation, considered in ([4]). Let us recall the corresponding definitions. An operator has finite memory if it can be specified by equations of the following form (canonical equations):

$$z(n) = \Phi(q(n), x(n), x(n+1), \ldots),$$

$$q(n+1) = \Psi(q(n), x(n), x(n+1), \ldots),$$

$$q(1) = q_0,$$

where $q(n)$ takes values from some auxiliary finite alphabet, called the alphabet of states. Otherwise the operator has infinite memory. Further, an operator has finite anticipation p if, for any n, the value $z(n)$ is uniquely determined by the values $x(1), \ldots, x(n+p)$. An operator with finite memory and finite anticipation is called finite.

Theorem 2. *The class of operators computable by A-machines is wider than the class of finite operators, but narrower than the class of all effective operators.*

The proof of Theorem 2 reduces to the justification of the following three assertions:

I. Every finite (in particular, automaton) operator is realizable by an A-machine.

The proof is trivial.

II. There exists an A-machine realizing a (constant) operator with infinite memory ([3]).

Such a machine is easy to construct, starting from the machine without erasure given in Table 1, which, beginning its work on a blank tape in state q_{00}, computes the nonperiodic sequence

$$101001 \ldots \underbrace{10 \ldots 01}_{2^n} \ldots$$

Table 1

	q_{00}	q_{01}	q_{02}	q_{03}	q_{04}	q_{05}	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{10}	q_{11}
1							rq_2	rq_9	rq_4					lq_1			
0							l	r	r					l		rq_9	
$\wedge$	1rq_{01}	0rq_{02}	1rq_{03}	rq_{04}	0rq_{05}	1lq_1	l	0q_3	r	rq_5	0rq_6	rq_7	0q_8	l	rq_{10}	lq_{11}	1lq_1

III. There exists an effective operator not realizable by any A-machine.

The input and output alphabets of this operator are, respectively, $\{0, 1\}$ and $\{a, b\}$, and it is given by the canonical equations ([4])

$$z(n) = q(n),$$

$$q(n+1) = f(q(n), x(n), x(n+1), \ldots), \tag{1}$$

$$q(1) = a$$

(the alphabet of states coincides with the output alphabet). f is defined as follows:

$$f(a, x(n), \ldots) = \begin{cases} b, & \text{if } x(n+1) = x(n+2) = 1, \\ a & \text{in all other cases.} \end{cases}$$

To determine the values of $f(b, x(n), \ldots)$, consider the sequence of words Δ_n, where $\Delta_1 = 11$, and Δ_{n+1} is obtained from Δ_n by deleting the first symbol and appending on the right the word

$$\underbrace{0 \ldots 0}_{n} 01.$$

Now the value $f(b, x(n), \ldots)$ is determined as follows. From the sequence $x(n), x(n+1) \ldots$ one selects the minimal initial segment coinciding with some Δ_m. If the sequence $x(n+1), x(n+2), \ldots$ begins with Δ_{m+1}, then $f(b, x(n), \ldots) = b$; otherwise $f(b, x(n), \ldots) = a$.

We note that although f is not defined everywhere, nevertheless the operator given by equations (1) is defined for all sequences of input letters. The constructed operator has finite memory, but does not have finite anticipation ([4]). Thus, incidentally, the following has been established:

Theorem 3. *The class of effective operators with finite memory is wider than the class of finite operators.*

3°. **Definition 2.** An A-machine is a **machine with finite signaling** if for some natural p the following holds. Whatever n may be, no tape cell with number greater than $n + p$ is scanned by the machine before the moment of computation of $z(n)$.

Theorem 4. *An A-machine with finite signaling realizes a finite operator.*

The proof of this theorem is based on the following definition and lemma.

Definition 3. Two words σ_1, σ_2 over the external alphabet of an A-machine T are **indistinguishable by this machine** if, at any moment of the operation of T, the initial segment of the tape filled with the word σ_1 and not containing the scanned cell can be replaced by a segment of tape filled with the word σ_2, and this will not affect the processing of the remaining part of the tape.

Lemma. *The set of words over the external alphabet of an A-machine that contain no more than p input letters (and any number of output letters) can be partitioned into a finite number of classes in such a way that words of the same class will be indistinguishable by the given A-machine.*

The assertion of the theorem follows from this lemma, since during the operation of an A-machine with signaling p, to the left of the scanned cell there cannot be more than p input symbols.

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Note: Figure translations are in progress. See original paper for figures.