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Abstract

Full Text

GEOPHYSICS

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COMPUTATION OF THE S_2 TIDAL COMPONENT FOR THE PACIFIC OCEAN AREA ON THE BESM-2 ELECTRONIC COMPUTER

(Presented by Academician D. I. Shcherbakov, 22 I 1963)

In recent years, numerical methods for solving the hydrodynamic equations of tides have been successfully applied in oceanographic practice to determine the harmonic constants of tides and tidal currents in the open sea. At present, tidal charts have been constructed for many seas by Hansen's method ⁽⁵⁾, and these charts adequately reflect the actual pattern of the distribution of tides and tidal currents.

As for ocean basins, there are only two studies in which tides and tidal currents are computed for the North Atlantic. The first of them was published in 1949 by Hansen ⁽⁵⁾, and it sets out the method of computation. In that work, however, only one constituent wave, M_2 , is considered, with very coarse detail. The computation was carried out only for 17 interior points on a grid with a spacing of 10° . The second work was published by L. I. Boris in 1961 ⁽³⁾. In it, tides and tidal currents in the North Atlantic are computed for two constituent waves, M_2 and S_2 , with more detailed resolution. The computation was carried out for 107 interior points on a grid with a spacing of 5° .

The application of Hansen's method, or of similar methods for the numerical solution of the hydrodynamic equations of tides, to ocean basins encounters a number of methodological and technical difficulties. First of all, difficulties arise in determining the initial values of the level on the open boundaries of the chosen contour. With a sufficiently large grid spacing, difficulties also arise in determining depths near the solid boundaries of the contour. Refinement of the grid in the interior region for greater detail of the phenomenon leads to a sharp increase in the number of equations to be calculated, and hence to an increase in the time and complexity of the computations, and does not improve accuracy because of the presence of open boundaries.

The present work represents the first attempt to compute the harmonic constants of the tidal constituent S_2 for the Pacific Ocean area as a whole. This wave was chosen first because, according to the available information, it has the most complex pattern of propagation over the Pacific Ocean area and, at the same time, a very small amplitude.

As the starting point, the differential equation of elliptic type ⁽⁵⁾ was taken:

$$\begin{aligned}
 & -\frac{i\sigma ab(1 + \alpha^2 \sin^2 \varphi)}{gh} \zeta - \zeta_{\varphi\varphi} - \frac{1}{\cos^2 \varphi} \zeta_{\lambda\lambda} - \\
 & -\zeta_{\varphi} \left(\frac{h_{\varphi}}{h} + \alpha \operatorname{tg} \varphi \frac{h_{\lambda}}{h} - \frac{1 + 2\alpha^2 - \alpha^2 \sin^2 \varphi}{1 + \alpha^2 \sin^2 \varphi} \operatorname{tg} \varphi \right) - \\
 & -\zeta_{\lambda} \left(-\alpha \frac{h_{\varphi}}{h} \operatorname{tg} \varphi + \frac{1}{\cos^2 \varphi} \frac{h_{\lambda}}{h} - \alpha \frac{1 - \alpha^2 \sin^2 \varphi}{1 + \alpha^2 \sin^2 \varphi} \right) + \zeta_{\varphi\varphi}^* + \frac{1}{\cos^2 \varphi} \zeta_{\lambda\lambda} + \\
 & + \zeta_{\varphi}^* \left(\frac{h_{\varphi}}{h} \alpha \operatorname{tg} \varphi \frac{h_{\lambda}}{h} - \frac{1 - 2\alpha^2 - \alpha^2 \sin^2 \varphi}{1 + \alpha^2 \sin^2 \varphi} \operatorname{tg} \varphi \right) + \\
 & + \zeta_{\lambda}^* \left(-\alpha \frac{h_{\varphi}}{h} \operatorname{tg} \varphi + \frac{h_{\lambda}}{h} \frac{1}{\cos^2 \varphi} - \alpha \frac{1 - \alpha^2 \sin^2 \varphi}{1 + \alpha^2 \sin^2 \varphi} \right) = 0,
 \end{aligned}$$

where ζ is the deviation of the level height from the mean; φ and λ are latitude and longitude; ω is the angular velocity of the Earth's rotation; a is the Earth's radius; σ is the angular...

wave speed, ρ is the coefficient of friction, h is the depth of the sea, $g\zeta^*$ is the tide-generating potential, g is the acceleration due to gravity, $a = \frac{2\omega}{\delta}$, $b = a\delta$, $\delta = -i\sigma + \rho$; $\rho = 0$ was adopted.

The tide-generating potential entering into the calculation was determined by the formula

$$\zeta^* = K \exp(-i\sigma t),$$

where K is the magnitude of the potential ⁽²⁾, and t is local time. This equation is solved uniquely under the conditions that $\rho \neq 0$ or $\sigma \neq 2\omega \sin \varphi$. In the present case the second condition is satisfied. To solve this equation by a numerical method, all partial derivatives in the equation were replaced by finite differences and, after the corresponding transformations, the following working equation was obtained:

$$\begin{aligned}
 Q_0 = \frac{1}{M} \{ & C_0 \zeta_0^* + h_0(Q_2 + Q_4) + h_0 C_1(Q_1 + Q_3) + (H_2 + C_2 h_0 + i C_3 H_1) \times \\
 & \times (Q_2 - Q_4) + (C_1 H_1 - i C_3 H_2 + i C_4 h_0)(Q_1 - Q_3) \},
 \end{aligned}$$

where $Q_i = \zeta_i^* - \zeta_i$, $M = C_0 + 2h_0(1 + C_1)$,

$$H_1 = \frac{1}{4}(h_1 - h_3), \quad H_2 = \frac{1}{4}(h_2 - h_4), \quad C_0 = l^2\gamma(-1 + \beta^2 \sin^2 \varphi),$$

$$C_1 = \frac{1}{\cos^2 \varphi}, \quad C_2 = \frac{l}{2} \operatorname{tg} \varphi (1 - 2\beta^2 + \beta^2 \sin^2 \varphi) : (-1 + \beta^2 \sin^2 \varphi),$$

$$C_3 = \beta \operatorname{tg} \varphi, \quad C_4 = \frac{l}{2} \beta (1 + \beta^2 \sin^2 \varphi) : (-1 + \sin^2 \varphi),$$

l is the grid step in radian measure, $\gamma = \frac{a^2 \delta^2}{g}$, $\beta = \frac{2\omega}{\delta}$.

This equation gives a relation between the values of the function at some interior point and the values of this function at four neighboring points in the interior region or on the boundary of the contour. Such an equation was written for each computational point of the interior region.

The computations were carried out for a grid with a step of 5° . The northern, eastern, and western boundaries of the grid almost coincide with the natural boundaries of the Pacific Ocean, while the southern boundary of the contour passes along latitude 60° S. The position of the southern boundary of the contour is due to the fact that on the coast of Antarctica in the Pacific sector there are only five points with harmonic tidal constants, which are situated practically in one place, in the western part of the Ross Sea. This circumstance does not permit reliable values of sea level to be specified on the coast of Antarctica. Cotidal charts constructed by the method of isogyres ⁽¹⁾ are also very inaccurate in this region for the same reason. It therefore proved expedient to take latitude 60° S as the southern boundary of the contour, where the previously constructed charts are more reliable than near the coast of Antarctica.

The grid consists of 633 points, of which 103 are boundary points, for which initial data were calculated. The initial data used were averaged values of the harmonic tidal constants for coastal stations, published in the Tide Tables ⁽⁴⁾. On the water boundary of the southern part of the contour, the initial values of the level were taken from a cotidal chart constructed by the method of isogyres ⁽¹⁾. In addition, the initial values for all boundaries of the contour were compared with the cotidal chart. The depths for each interior point were calculated as averages for the five-degree square at whose center the given point is located. In regions of numerous small islands the depths were determined without taking them into account, but with allowance for the actual depths near the islands.

In the open part of the ocean, five points were selected near islands that have actual harmonic tidal constants, which served, as it were, as boundary points. These points were not included in the computation when solving the prob-

Fig. 1. Tidal chart of the wave S_2 . Cotidal lines are shown by solid lines, isoamplitudes by dashed lines, and the computational grid is indicated by dots.

Figure 1: Fig. 1. Tidal chart of the wave S_2 . Cotidal lines are shown by solid lines, isoamplitudes by dashed lines, and the computational grid is indicated by dots.

task, but the calculation was carried out taking into account the actual tidal values at these points. The exclusion of these points from the interior domain was necessary for the stability of the iterative method, which depends on the logarithmic derivative of the depth.

Fig. 1. Tidal chart of the wave S_2 . Cotidal lines are shown by solid lines, isoamplitudes by dashed lines, and the computational grid is indicated by dots.

The question of the stability of the iterative method causes great difficulties. The conditions for satisfying the stability criteria were checked for model equations, and it was found that, in order to satisfy the stability conditions, the logarithmic derivative of the depth must not be large. This circumstance was confirmed in the calculations. The calculation of the wave S_2 for the Pacific Ocean area and a control calculation for the North Atlantic showed that the convergence of the iterations has an asymptotic character: the differences between the values of successive iterations decrease from 32 to 0.5 cm, and then oscillatory instability appears.

The islands of New Zealand were also approximated by four boundary points, since they cannot be neglected owing to the complex propagation of the tidal wave in this region. The introduction of such points into the internal computational domain is entirely correct and does not change the essence of the problem being solved.

The system of the above-mentioned 633 linear algebraic equations, after standard programming, was solved by the method of iterations on the BESM-2 electronic computer. For the complete solution of this problem, after debugging of the program, 15 minutes of machine time were required.

Figure 1 gives the tidal chart of the constituent tide S_2 , constructed from the calculated data. It is evident from it that the propagation of this wave

over the water area of the Pacific Ocean has a very complex character. Within the ocean area there are six clearly expressed amphidromic systems and two nodal zones to the north and to the east of the island of New Guinea.

The tidal chart presented agrees well with the actual data on tidal fluctuations of level on the coast and islands of the Pacific Ocean and differs hardly at all from an analogous chart constructed by another method ⁽¹⁾.

At present, analogous computations of the constituent tidal waves M_2 , K_1 , and O_1 on the BESM-2 electronic computer are being completed, and they will be

published soon.

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Note: Figure translations are in progress. See original paper for figures.

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