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CYBERNETICS AND CONTROL THEORY

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Abstract

Full Text

CYBERNETICS AND CONTROL THEORY

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A PRINCIPLE FOR CONSTRUCTING COMBINED AUTOMATIC CONTROL SYSTEMS WITH VARIABLE STRUCTURE

In a number of cases, in order to improve the static and dynamic accuracy of reproducing a control action, combined tracking systems are used. Transient processes in such systems can be described by a linear nonhomogeneous differential equation

$$M(p)x = N(p)g(t), \quad (1)$$

where $M(p)$, $N(p)$ are operator polynomials with respect to p ; $p \equiv d/dt$; x is the error signal; $g(t)$ is the control action, which is a continuous function of time.

The independence of the error signal x from the control action $g(t)$ is usually determined by the condition

$$N(p) = 0. \quad (2)$$

In this case the forced component $x_{\text{forced}}(t)$ of the general solution of equation (1) is identically equal to zero.

In the general case, to satisfy (2) it is necessary: 1) to differentiate the control action, 2) to keep the parameters of the open and closed loops of the combined tracking system unchanged. We shall try to solve this problem without requiring condition (2) to be fulfilled, thereby weakening the restrictions 1), 2) stated above.

Let, in a domain G of the n -dimensional space x_1, \dots, x_n , the motion of a dynamic system be described by a system of nonhomogeneous differential equations

$$dx/dt = \mathbf{f}(\mathbf{x}, \mathbf{g}(t)), \quad (3)$$

where $\mathbf{x} = (x_1, \dots, x_n)$; $\mathbf{f} = (f_1, \dots, f_n)$; $f_i = x_{i+1}$ ($i = 1, 2, \dots, n-1$);

$$f_n = -\sum_{i=1}^n a_i x_i + \sum_{i=1}^m \psi_i(\mathbf{x}) g_i(t);$$

$$\psi_i(\mathbf{x}) = \begin{cases} b_i^1, & \text{for } \sum_{j=1}^n c_j x_j > 0, \\ b_i^2, & \text{for } \sum_{j=1}^n c_j x_j < 0, \end{cases} \quad (i = 1, 2, \dots, n);$$

a_i, b_i^1, b_i^2, c_j are constant quantities; $g_i(t)$ is a specified function continuous over the entire time interval t .

Let the hyperplane S , given by the equation $\sum_{j=1}^n c_j x_j = 0$, divide the domain G into the domains G^+ ($\sum_{j=1}^n c_j x_j > 0$) and G^- ($\sum_{j=1}^n c_j x_j < 0$), in which the vector-function $\mathbf{f}(\mathbf{x}, \mathbf{g}(t))$ of system (3) is bounded, and for any constant value of time t , as S is approached from G^+ and G^- , its limiting values \mathbf{f}^+ and \mathbf{f}^- exist. Suppose that, as the solution $\mathbf{x}(t)$ approaches some domain $U \subset S$, the vector-functions \mathbf{f}^+ and \mathbf{f}^- are directed toward the hyperplane S ($f_N^-(\mathbf{x}) \geq 0, f_N^+(\mathbf{x}) \leq 0, f_N^- - f_N^+ > 0$, where f_N^+ and f_N^- are the projections of the vectors \mathbf{f}^+ and \mathbf{f}^- onto the normal to the hyperplane S , directed from G^- to G^+). Then, for

* In the case $\sum_{j=1}^n c_j x_j = 0, \quad \psi_i(\mathbf{x}) = b_i^1$ for $\sum_{j=1}^n c_j x_j \rightarrow +0; \quad \psi_i(\mathbf{x}) = b_i^2$ for $\sum_{j=1}^n c_j x_j \rightarrow -0$.

when $\mathbf{x}(t)$ enters U , a so-called "sliding mode" arises, and the solution of system (3) does not depend on $a_i, b_i^1, b_i^2, g_i(t)$.

Indeed, in this case, as shown in the work of A. F. Filippov [1], in the region U there exists a solution $\mathbf{x}(t)$ of system (3), and the vector $d\mathbf{x}/dt = \mathbf{f}^0(\mathbf{x})$ lies in the hyperplane S and is determined by the values of the vector functions \mathbf{f}^+ and \mathbf{f}^- . From the condition that $\mathbf{f}^0 \in S$, there follows a linear dependence of the components of the vector \mathbf{f}^0 :

$$\sum_{j=1}^n c_j f_j^0 = 0, \quad (4)$$

where f_j^0 is the j -th component of the vector \mathbf{f}^0 , whence

$$f_n^0 = -\frac{1}{c_n} \sum_{j=1}^{n-1} c_j f_j^0. \quad (5)$$

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Fig. 1

Consequently, the solution of system (3) for $\mathbf{x}(t) \in U$ coincides with the solution of the linear system of differential equations

$$d\mathbf{x}/dt = \mathbf{f}^0(\mathbf{x}), \tag{6}$$

where $\mathbf{x} = (x_1, \dots, x_n)$; $\mathbf{f}^0 = (f_1^0, \dots, f_n^0)$; $f_i^0 = x_{j+1}$ ($j = 1, 2, \dots, n - 1$);

$$f_n^0 = -\frac{1}{c_n} \sum_{j=1}^{n-1} c_j x_{j+1};$$

c_j are constant quantities. It is obvious that the solution of system (6) does not depend on $a_i, b_i^1, b_i^2, g_i(t)$.

Fig. 2

We shall construct a combined tracking system in such a way that, when the parameters of the closed and open loops and the various functions of the control action $g(t)$ are varied within sufficiently wide limits: 1) the region U exists, includes the origin, and the solution of the system of differential equations (6) satisfies the specified requirements for the quality of the control process (the control time and the maximum dynamic error of the system must not exceed certain prescribed values); 2) for any initial conditions the solution of system (3) enters the region U , and the coordinate x_1 changes sign no more than once; 3) in the region U there are no trajectories serving as sections of limit cycles with a partial sliding mode. Then the solution of the original system of differential equations (3) will depend on the control disturbance and on the parameters of the closed and open loops only up to the instant at which $\mathbf{x}(t)$ enters the region U , where the solution coincides with the solution of the system of linear homogeneous differenti-

al equations (6). Since the transient process ends in the region U , the combined follow-up system reproduces a broad class of control actions $g(t)$ without static errors (the error x_1 of the follow-up system is invariant with respect to $g(t)$, up to the transient component), and since the solution in U depends only on the coefficients c_j , the influence of the parameters of the closed and open loops on the quality of the control process is substantially weakened.

Fig. 3

Figure 3: Fig. 3

Fig. 3

Example. Let the combined follow-up system (Fig. 1) be described by a system of nonhomogeneous differential equations with discontinuous right-hand side

$$dx/dt = f(x, g(t)), \quad (7)$$

where

$$x = (x_1, x_2); \quad g = (g_1, g_2, g_3); \quad f = (f_1, f_2); \quad f_1 = x_2,$$

$$f_2 = -2bx_2 - \omega_0^2 x_1 + g_3(t) + 2bg_2(t) + \Phi(x, g_1(t))g_1(t);$$

$$2b = \frac{T_1 + T_2}{T_1 T_2}; \quad \omega_0^2 = \frac{1 + k_1 k_2}{T_1 T_2}; \quad g_3(t) = \frac{dg_2(t)}{dt}; \quad g_2(t) = \frac{dg_1(t)}{dt};$$

$$\Phi_1(x, g_1(t)) = \frac{1 + \Phi_1^*(x, g_1(t))}{T_1 T_2};$$

$$\Phi_1^*(x, g_1(t)) = \begin{cases} +K, & \text{when } \left(\sum_{j=1}^2 c_{jxj} \right) g_1(t) < 0, \\ -K, & \text{when } \left(\sum_{j=1}^2 c_{jxj} \right) g_1(t) > 0^*; \end{cases}$$

$k_1, k_2, T_1, T_2, c_1, c_2$ are constant quantities.

* When $\left(\sum_{j=1}^2 c_{jxj} \right) g_1(t) = 0$, $\Phi_1^*(x, g_1(t)) = +K$ when $\left(\sum_{j=1}^2 c_{jxj} \right) g_1(t) \rightarrow +0$,

$$\Phi_1^*(x, g_1(t)) = -K \quad \text{when } \left(\sum_{j=1}^2 c_{jxj} \right) g_1(t) \rightarrow -0.$$

Let $g_1(t) = A$, where A is a constant, and suppose that the parameters of the tracking system k_1, k_2, T_1, T_2, K are chosen so that the conditions

$$Kk_2 > 1; \quad b^2 > \omega_0^2; \quad c_1/c_2 = b - \sqrt{b^2 - \omega_0^2}. \quad (8)$$

are satisfied.

Then for $g_1(t) > 0$ the phase plane of the system will have the form shown in Fig. 2 *a, b, c*. In this case, when the depicting point falls on the line $c_1x_1 + c_2x_2 = 0$, the solution of equation (7) coincides with the solution of the system of linear homogeneous equations

$$d\mathbf{x}/dt = \mathbf{f}^0(\mathbf{x}), \quad (9)$$

where $\mathbf{x} = (x_1, x_2)$; $\mathbf{f}^0 = (f_1^0, f_2^0)$; $f_1^0 = x_2$; $f_2^0 = -\frac{c_1}{c_2}x_2$. From (9) it follows that the steady-state reproduction error $g(t) = A$ will be absent.

Let the parameters of the system change so that the inequality $b^2 > \omega_0^2$ is transformed into the inequality $b^2 < \omega_0^2$. The phase plane of the system will change and will take the form shown in Fig. 3 *a, b, c*. In this case, as in the preceding one, the transient process will proceed without overshoot, and the steady-state error will be absent. If the parameters of the closed loop change so that without the open loop the system becomes unstable—for example, the sign before $2bx_2$ in equation (7) changes—then, as follows from the phase plane (Fig. 3 *g, d, e*), the dynamic and static properties of the system will practically not differ from the case considered above. When reproducing other types of control actions, for example $g_1(t) = Ae^{\alpha_1 t}$, where α_1, A are constants (Fig. 4 *a, b*), for combinations of parameters satisfying the conditions

$$Kk_2 > 1; \quad b^2 < \omega_0^2; \quad c_1/c_2 = b - \sqrt{b^2 - \omega_0^2}, \quad (10)$$

the vector functions \mathbf{f}^+ and \mathbf{f}^- at any instant of time are such that the depicting point, under any initial conditions, falls on the line $c_1x_1 + c_2x_2 = 0$. Consequently, even when reproducing a control action specified in the form of a transcendental function, the steady-state error is absent.

Fig. 4

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1. A. F. Filippov, *Matem. sborn.*, **51**, No. 1 (1960).

Note: Figure translations are in progress. See original paper for figures.

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