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# MATHEMATICS

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**Abstract**

**Full Text**

## MATHEMATICS

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### FIBRATION OF TWO-PARAMETER FAMILIES OF STRAIGHT LINES BY LINES IN A MULTIDIMENSIONAL PROJECTIVE SPACE

*(Presented by Academician P. S. Novikov, 24 X 1962)*

The fibration of manifolds by families of lines was proposed in 1945 by S. P. Finikov and applied to the construction of a fibered pair of line-like surfaces <sup>(1)</sup> and complexes of straight lines <sup>(2)</sup>. In our article <sup>(3)</sup> this notion was generalized to pairs of manifolds with generating elements of different dimension in four-dimensional projective space. In the present work we carry out the fibration by families of lines of a pair of two-parameter families of straight lines in  $n$ -dimensional projective space  $P_n$  ( $n > 3$ ).

1. In the space  $P_n$  consider a pair of families  $\infty^2$  of straight lines  $\{l\}$  and  $\{l'\}$ , whose elements are in one-to-one correspondence. With each straight line  $l$  of one family we associate a pencil of plane elements (two-dimensional planes), the centers of which lie on  $l$ , while the axis of the pencil is  $l'$ . We shall call the pair  $\{l, l'\}$  *one-sidedly fibered* in the direction from  $\{l\}$  to  $\{l'\}$  if the plane elements are tangent planes to the lines described by the centers of these elements as  $l$  moves within the subfamily  $\{l\}$ . The lines described by the centers of the plane elements will be called the *fibering lines*. If the pair  $\{l, l'\}$  is fibered both in the direction from  $\{l\}$  to  $\{l'\}$  and in the direction from  $\{l'\}$  to  $\{l\}$ , then such a pair will be called *two-sidedly fibered*, or simply a *fibered pair*.

We shall assume that the corresponding elements of the families  $\{l\}$ ,  $\{l'\}$  do not intersect and that the family  $\{l\}$  is focal, possessing two distinct focal surfaces.

2. Assuming that *only one focal plane of the ray  $l$  intersects the ray  $l'$* , place the vertices  $A_1$  and  $A_2$  of a moving frame at the foci of the ray  $l$ , the vertex  $A_3$  at the point of intersection of one of the focal planes of the ray  $l$  with the ray  $l'$ , the vertex  $A_4$  arbitrarily on  $l'$ , and the vertex  $A_5$  in the second focal plane of the ray  $l$ . We do not fix the positions of the remaining vertices  $A_\alpha$  ( $\alpha = 6, 7, \dots, n + 1$ ) of the frame. The infinitesimal displacements of the vertices of the frame are determined by the equations  $dA_i = \omega_i^j A_j$ , where the forms  $\omega_i^j$  satisfy the structure equations of projective space  $D\omega_i^j = [\omega_i^k \omega_k^j]$  ( $i, j, k = 1, 2, \dots, n + 1$ ). With the indicated choice of the

vertices of the frame, the family  $\{l\}$  is determined by the equations

$$\omega_1^4 = \omega_1^5 = \omega_1^\alpha = \omega_2^3 = \omega_2^4 = \omega_2^\alpha = 0, \quad [\omega_1^3 \omega_2^5] \neq 0 \quad (1)$$

and depends on an arbitrary two functions of two arguments. The developable surfaces of the family are determined by the equations  $\omega_1^3 = 0$  and  $\omega_2^5 = 0$ . The tangent plane of the focal surface ( $A_1$ ) intersects the ray  $l'$  at the point  $A_3$ .

The point  $M = A_1 + \rho A_2$  describes a fibering line ( $M$ ) if along this line

$$(dM, M, A_3, A_4) = 0, \quad (d^2M, M, A_3, A_4) = 0.$$

The first of these conditions gives the equations of the ruling lines ( $M$ ):

$$\omega_2^5 = 0, \quad d\rho + \rho(\omega_2^2 - \omega_1^1) + \omega_1^2 - \rho^2 \omega_2^1 = 0,$$

the second gives the conditions of a one-sided fibration:

$$\omega_3^1 \equiv 0, \quad \omega_3^2 \equiv 0, \quad \omega_3^5 \equiv 0, \quad \omega_3^\alpha \equiv 0 \pmod{\omega_2^5}. \quad (2)$$

The point  $M' = A_3 + \rho' A_4$  describes a ruling line ( $M'$ ), if along this line

$$(dM', M', A_1, A_2) = 0, \quad (d^2M', M', A_1, A_2) = 0.$$

Whence, in addition to equations (2), we obtain

$$\omega_4^1 = 0, \quad \omega_4^5 \equiv 0, \quad \omega_4^\alpha = 0 \pmod{\omega_2^5}, \quad (3)$$

and the ruling lines ( $M'$ ) are determined by the equations

$$\omega_2^5 = 0, \quad d\rho' + \rho'(\omega_4^4 - \omega_3^3) + \omega_3^4 - (\rho')^2 \omega_4^3 = 0.$$

Equations (1), (2), (3) and their exterior differentials constitute the system of equations of the problem.

**Theorem 1.** *A fiberable pair  $\{l, l'\}$  exists with an arbitrary number  $\varepsilon$ : two functions of two arguments.*

By virtue of the one-sided fibration, the family  $\{l'\}$  becomes focal: the unique (without additional restrictions) focal surface is described by the point  $A_3$  of intersection of one of the focal planes of the ray  $l$  with the ray  $l'$ ; the developable surface is defined by the equation  $\omega_2^5 = 0$ . The families of ruling lines ( $M$ ) and ( $M'$ ) correspond and lie in the developable surfaces of the families  $\{l\}$ ,  $\{l'\}$ . The lines ( $M$ ), ( $M'$ ) are three-dimensional, belong to the space stretched over the

lines  $l, l'$ . The tangents to the ruling lines ( $M$ ) at the points of the ray  $l$  pass through the focus  $A_3$  of the ray  $l'$ , while the tangents to the ruling lines ( $M'$ ) at the points of the ray  $l'$  pass through the focus  $A_2$  of the ray  $l$ .

**3. Theorem 2.** *If both focal planes of the ray  $l$  intersect the ray  $l'$ , then under a one-sided fibration in the direction from  $\{l\}$  to  $\{l'\}$ , either the pair  $\{l, l'\}$  is immersed in a three-dimensional space, or the ruling lines ( $M$ ) become straight lines.*

**4. Theorem 3.** *If both focal planes of the ray  $l$  do not intersect the ray  $l'$ , then the ruling lines ( $M$ ) do not exist.*

5. Let us summarize the above in the form of a theorem:

**Theorem 4.** *Let  $\{l\}$  and  $\{l'\}$  be two-parameter families of lines in  $P_n$ , and let  $\{l\}$  possess two distinct focal surfaces. A fibration of the pair  $\{l, l'\}$  by families of lines in  $P_n$  ( $n > 3$ ) is possible if and only if only one focal plane of the ray  $l$  intersects the ray  $l'$ .*

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## CITED LITERATURE

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*Note: Figure translations are in progress. See original paper for figures.*

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