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Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Abstract

Full Text

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MOTION OF A FREE GYROSCOPE ON A MOVING BASE, TAKING INTO ACCOUNT THE MASSES OF THE RINGS OF THE CARDAN SUSPENSION

(Presented by Academician A. Yu. Ishlinskii, 30 III 1963)

The motion of a free gyroscope in a Cardan suspension on a moving base has been considered in a number of works ([1−4]), in which it was assumed that the deviations of the gyroscope axis and of the rings are small, as a result of which, in the equations of motion, small quantities no higher than the second order were retained. In the present work, in deriving the general equations, the assumption of smallness of the angles is discarded. However, in solving particular problems it is assumed that the angles of deviation of the gyroscope axis from its initial position in fixed space are small (this is the case in particular instruments), while the angles of deviation of the rings are regarded as finite. The equations obtained are used in this work to compute the deviation of the gyroscope axis in an inertial coordinate system as a result of slow rotations of the base over a long time interval (the drift of a gyroscope on the Earth over a day).

Fig. 1

Fig. 2

1. Coordinate systems. Let us turn directly to the system whose motion we are investigating, and to the coordinates of this system. The position of the gyroscope axis can be determined by the angles α and β, as shown in Fig. 1. It is assumed that at the initial instant the gyroscope axis coincides with the x-axis (x, y, z is a fixed coordinate system). After this, in order to determine the position of the casing it is sufficient to introduce the angle of rotation γ about the gyroscope axis. The same drawing shows the angle δ_1—the angle of rotation of the outer frame relative to the casing. Let ψ, ϑ, φ be the angles determining

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Fig. 3

Figure 3: Fig. 3

the position of the base; δ_2 the angle of rotation of the outer frame with respect to the base (Fig. 2). It is clear that any 5 of the 8 coordinates introduced determine the position of the system, while three coordinates are superfluous. Nevertheless, we shall take as the coordinates of the system the 8 quantities $\alpha, \beta, \gamma, \delta_1, \delta_2, \psi, \vartheta, \varphi$, of which

the first two determine the drift of the gyroscope, while the last three determine the position of the moving base (motion about a fixed point is meant).

2. Kinematic relations. Obviously, the 8 coordinates are not independent. Relations can be established between them by writing, for example, that the angular velocity of the outer frame, expressed through $\alpha, \beta, \gamma, \delta_1$, on the one hand,

$$\bar{\Omega}_p = \bar{\alpha} + \bar{\beta} + \bar{\gamma} + \bar{\delta}_1, \tag{2,1}$$

and through $\psi, \vartheta, \varphi, \delta_2$, on the other,

$$\bar{\Omega}_p = \bar{\psi} + \bar{\vartheta} + \bar{\varphi} + \bar{\delta}_2, \tag{2,2}$$

has one and the same value. Expressing the right-hand sides of equalities (2,1) and (2,2) in terms of the unit vectors of the intermediate coordinate systems (they are shown in Fig. 1) and then in terms of the unit vectors of the systems with index 4 (the systems x_4, y_4, z_4 and x_4', y_4', z_4' are identical—both are connected with the outer frame), we obtain a vector equality. Equating the coefficients of like unit vectors, we find three kinematic relations, which give the desired connection between the coordinates $\alpha, \beta, \gamma, \delta_1, \delta_2, \psi, \vartheta, \varphi$ and their derivatives

Fig. 3

$$\begin{aligned} \dot{\gamma} - \dot{\alpha} \sin\beta + \dot{\delta}_2 \sin\delta_1 &= \bar{p}_0 \cos\delta_1 - \bar{q}_0 \sin\delta_1, \\ \dot{\alpha} \sin\gamma + \dot{\beta} \cos\gamma - \dot{\delta}_2 \cos\delta_1 &= \bar{p}_0 \sin\delta_1 + \bar{q}_0 \cos\delta_1, \\ \dot{\alpha} \cos\gamma \cos\beta - \dot{\beta} \sin\gamma + \dot{\delta}_1 &= \bar{r}_0. \end{aligned} \tag{2,3}$$

Here

$$\bar{p}_0 = p_0 \cos\delta_2 - r_0 \sin\delta_2, \qquad \bar{q}_0 = q_0, \qquad \bar{r}_0 = p_0 \sin\delta_2 + r_0 \cos\delta_2,$$

and $p_0,\ q_0,\ r_0$ are the projections of the angular velocity of the base onto axes fixed to it.

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3. Kinetic energy. The kinetic energy of the system can be represented in the form

$$T = T_1 + T_2 + T_3 + T_4, \tag{3,1}$$

where T_1 is the kinetic energy of the gyroscope. The indices 2, 3, 4 refer to the casing, the outer frame, and the base, respectively. The kinetic energy of the gyroscope can be expressed through α, β, ω (the angular velocity of its own rotation), and the kinetic energy of the casing and the frame through $\delta_1, \delta_2, \psi, \vartheta, \varphi$. Let us note that γ will not enter into these expressions.

$$T_1 = \frac{1}{2}\{A_1(\omega - \dot{\alpha}\sin\beta)^2 + B_1\dot{\beta}^2 + C_1\dot{\alpha}^2\cos^2\beta\},$$

$$\begin{aligned} T_2 = \frac{1}{2}\{&A_2[(p_0\cos\delta_2 - r_0\sin\delta_2)\cos\delta_1 - (q_0 + \dot{\delta}_2)\sin\delta_1]^2 + \\ &+ B_2[(p_0\cos\delta_2 - r_0\sin\delta_2)\sin\delta_1 + (q_0 + \dot{\delta}_2)\cos\delta_1]^2 + \\ &+ C_2[(p_0\sin\delta_2 + r_0\cos\delta_2 - \dot{\delta}_1)]^2\}, \end{aligned} \tag{3,2}$$

$$T_3 = \frac{1}{2}[A_3(p_0\cos\delta_2 - r_0\sin\delta_2)^2 + B_3(q_0 + \dot{\delta}_2)^2 + C_3(p_0\sin\delta_1 + r_0\cos\delta_2)^2].$$

T_4 will depend only on the coordinates ψ, ϑ, φ and their derivatives. Since these coordinates will be assumed to be prescribed functions of time, co-

the corresponding equations are not needed, and therefore there is no need to compute T_4.

4. Equations of motion.

The position of the entire system together with the base can be determined by five coordinates $\alpha, \beta, \gamma, \delta_1, \delta_2$. Then the coordinates ψ, ϑ, φ will be superfluous. We have three constraint equations (2,3).

With a view to writing the equations of motion in Lagrange form, denote

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}}\right) - \frac{\partial T}{\partial \alpha} - Q_\alpha \equiv M_\alpha, \qquad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\beta}}\right) - \frac{\partial T}{\partial \beta} - Q_\beta \equiv M_\beta. \tag{4,1}$$

For the coordinates $\gamma, \delta_1, \delta_2$ we introduce analogous notation. Here Q_α, Q_β are generalized active forces. After these notations, the Lagrange equations with multipliers for the coordinates $\alpha, \beta, \gamma, \delta_1, \delta_2$ will have the form

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Fig. 4

Figure 4: Fig. 4

$$\begin{aligned} M_\alpha &= -\lambda_1 \sin\beta + \lambda_2 \cos\beta \sin\gamma + \lambda_3 \cos\beta\cos\gamma, \\ M_\beta &= \lambda_2 \cos\gamma - \lambda_3 \sin\gamma, \\ M_\gamma &= \lambda_1, \\ M_{\delta_1} &= \lambda_3, \\ M_{\delta_2} &= \lambda_1 \sin\delta_1 - \lambda_2 \cos\delta_1. \end{aligned} \tag{4,2}$$

In these equations $\lambda_1, \lambda_2, \lambda_3$ are undetermined multipliers. Since γ does not enter the expression for the kinetic energy and $Q_\gamma = 0$ (an ideal gyroscope is being considered), it follows that λ_1 is equal to zero. Eliminating $\lambda_1, \lambda_2, \lambda_3$ from equations (4,2), we obtain

$$\begin{aligned} M_\alpha &= -M_{\delta_2} \frac{\cos\beta}{\cos\delta_1} \sin\gamma + M_{\delta_1} \cos\gamma, \\ M_\beta &= -M_{\delta_2} \frac{1}{\cos\delta_1} \cos\gamma - M_{\delta_1} \sin\gamma. \end{aligned} \tag{4,3}$$

Write these equations, explicitly introducing the angles α and β (i.e. computing M_α and M_β). We shall compute M_{δ_1} and M_{δ_2} separately for each problem.

$$(A_1 \sin^2\beta + C_1 \cos^2\beta)\ddot{\alpha} + (A_1 - C_1)\sin 2\beta \cdot \dot{\alpha}\dot{\beta} - H\dot{\beta} - Q_\alpha =$$

$$= -M_{\delta_2} \frac{\cos\beta}{\cos\delta_1} \sin\gamma + M_{\delta_1} \cos\gamma, \tag{4,4}$$

$$B_1\ddot{\beta} + \frac{1}{2}(C_1 - A_1)\sin 2\beta\, \dot{\alpha}^2 + H\dot{\alpha} - Q_\beta = -M_{\delta_2} \frac{1}{\cos\delta_1} \cos\gamma - M_{\delta_1} \sin\gamma.$$

Fig. 4

For a gyroscope on a fixed base, computation of the constant drift velocity with the aid of equations (4,4) gives the formula

$$\dot{\delta}_2 = \frac{(A_2 + A_3)\sin\delta_1^0}{2H\cos^2\delta_1^0}\, \dot{\delta}_2^{0\,2},$$

which coincides with the result in Magnus' s article ([5]). Here δ_1^0, δ_2^0 are the initial values of the angles δ_1, δ_2; $H = A_1\omega\cos\beta$.

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5. A gyroscope on a rotating Earth.

Let the base of the gyroscope be located horizontally on the Earth. Denote the angular velocity of the Earth by Ω, and the latitude of the locality by θ. The coordinate axes connected with the base are arranged so that the x-axis is tangent to the parallel, and the y-axis is tangent to the meridian. At the initial instant these axes coincide with a fixed coordinate system, relative to which the deviation of the gyroscope axis (initially directed along the x-axis) over a day will be computed.

The velocity of motion of the base caused by the rotation of the Earth will have the following projections on the axes fixed to the base:

$$\begin{aligned} p_0 &= 0, \\ q_0 &= \Omega \cos\theta = U_2, \\ r_0 &= \Omega \sin\theta = U_1. \end{aligned} \tag{5,1}$$

In final form the equations determining the motion of the system will be:

$$\begin{aligned} -H\dot{\beta} &= \lambda_2 \sin\gamma, \\ H\dot{\alpha} &= \lambda_2 \cos\gamma, \\ \lambda_2 &= \frac{A_2 \sin\delta_1 \ddot{\gamma} - B_3 \ddot{\delta}_2 + (A_3 - C_3) U_1^2 \sin\delta_2 \cos\delta_2}{\cos\delta_1}, \\ \dot{\gamma} &= -U_1 \sin\delta_2 (\cos\delta_2)^{-1}, \\ -\dot{\delta}_2 &= -U_1 \sin\delta_2 \ \mathrm{tg}\,\delta_1 + U_2, \\ \dot{\delta}_1 &= U_1 \cos\delta_2 - \frac{\lambda_2}{H}. \end{aligned} \tag{5,2}$$

In the left-hand sides of these equations, small quantities above the second order have been discarded; friction is not taken into account. The solution of this system by means of a computing machine has the form shown in Figs. 3 and 4. For the constants entering the equations the following values were adopted: $A_1 = 40$ g·cm·sec^2, $A_2 = 10$ g·cm·sec^2, $A_3 = 10$ g·cm·sec^2, $C_3 = 8$ g·cm·sec^2, $B_3 = 7$ g·cm·sec^2, $\omega = 1250$ sec^{-1}, $\Omega = 7 \cdot 10^{-5}$ rad/sec, $\theta = 60^\circ$.

6. Solution of the system for $\theta < 30^\circ$. In this case one may put $\sin\theta = \mathrm{tg}\,\theta = \theta$ and in the 5th equation of system (5,2) neglect the first term, while in the 6th discard λ_2/H. After this, approximate integration of equations (5,2) gives, for the velocities of the nonperiodic drifts, the formulas:

$$\dot{\beta} = \frac{(2B_3 + A_2 - A_3 + C_3)B_3}{8H^2}\,\theta^4\Omega^3, \qquad -\dot{\alpha} = \frac{(2B_3 + A_2 - A_3 + C_3)B_3}{4H^2}\,\theta^5\Omega^3. \tag{5,3}$$

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