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Abstract

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A NUMERICAL METHOD FOR CALCULATING THE PROPAGATION OF LONG WAVES IN OPEN CHANNELS AND ITS APPLICATION TO THE FLOOD PROBLEM

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1. The very laborious calculations of unsteady motion of water flows in river systems became feasible only with the advent of electronic computing machines. However, forecasting the propagation of waves along a river channel with the aid of electronic computers has still not found wide application. This may be explained in part by the fact that, until recently, convenient numerical methods were lacking for which the expenditure of machine time would be relatively small. To solve the one-dimensional equations of unsteady motion in open channels, the authors have applied the method of grids. The difference implicit scheme developed makes it possible to carry out computations with a large time step. This is very important for calculations of floods in large rivers, when the duration of passage of a single flood wave is measured in weeks.

2. Unsteady, slowly varying motion of a liquid in an open channel is described by the Saint-Venant equations. For the numerical calculation of flows without discontinuities, the following form of the equations appears most convenient:

$$Bz_t + Q_x = q, \tag{1}$$

$$(1 - \mathrm{Fr})z_x + \frac{1}{g\omega}\left(\dot{Q}_t + 2vQ_x\right) = F, \qquad F = \left(i + \frac{\omega_x}{B}\right)\mathrm{Fr} - \frac{Q|Q|}{K^2}, \tag{2}$$

where x is the coordinate of the cross section; t is time; $z(x, h)$ is the ordinate of the free surface; $Q(x, h)$ is the discharge; $v(x, h)$ is the mean velocity; $\omega(x, h)$ is the cross-sectional area; $B(x, h)$ is the width of the free surface; $h(x, h)$ is the depth; $q(x,h)$ is the lateral inflow per unit length of channel; g is the acceleration of gravity; $i(x)$ is the bed slope; $K=\omega C\sqrt{R}$ is the discharge modulus; C is the Chezy coefficient; R is the hydraulic radius; $\mathrm{Fr}=(v/c)^2$ is the Froude number; $c=\sqrt{g\omega/B}$ is the speed of propagation of small disturbances; $\omega_x=\partial\omega/\partial x$ for $h=\mathrm{const}$.

We did not consider it expedient to discard the inertial terms in the Saint-Venant equations, as is done in some approximate methods. The presence of these terms cannot substantially complicate or increase the duration of a calculation carried out on an electronic computer. At the same time, the role of the inertial terms may become decisive when the discharge changes sharply with time, or, for example, when a flood wave enters a reservoir. We encounter a similar phenomenon in the calculation of certain navigation structures. A solution without neglecting the inertial terms makes it possible to consider a more varied range of problems.

For calculation at boundary points and at junction points, the equations are written in characteristic form:

$$[Q_t+(v\mp c)Q_x]-B(v\mp c)\left[z_t+(v\pm c)z_x\right]=Bc^2F-(v\mp c)q. \tag{3}$$

In calculating subcritical (tranquil) flows ($\mathrm{Fr}<1$) it was found that the usual implicit difference scheme (Fig. 1), in which all coefficien-

quantities in the derivatives and the right-hand sides of the equations are taken at the point n,k, leads to significant restrictions on the time step. For a qualitative analysis of this circumstance, entirely connected with equation (2), let us discard in it the terms that are usually inessential for $\mathrm{Fr}\ll 1$. Then the main features of the numerical solution of equation (2) can be reproduced on the "model" equation

$$\frac{\partial Q}{\partial t}=g\omega\left(I-\frac{Q^2}{K^2}\right),\qquad I=-\frac{\partial z}{\partial x}. \tag{4}$$

Let us restrict ourselves to the case of constant coefficients. The difference scheme indicated above for the system (1), (2) is analogous to the following scheme for equation (4) (Euler' s method):

$$\frac{Q_{k+1}-Q_k}{\tau}=g\omega\left(I-\frac{Q_k^2}{K^2}\right), \tag{5}$$

where τ is the time step. To clarify the question of the stability of the difference scheme, we linearize equation (5), putting

$$Q_k=K\sqrt{I}+y_k,$$

Fig. 1

Figure 1: Fig. 1

where y_k is a small quantity. Neglecting squares of small quantities, we obtain from (5) the linear equation

$$\frac{y_{k+1}-y_k}{\tau}+\frac{2g\omega\sqrt{I}}{K}\,y_k=0. \tag{6}$$

Fig. 1

A necessary condition for the stability of the computational process in solving equation (5) is the condition

$$\tau<\frac{K}{g\omega\sqrt{I}}=\frac{C}{g}\sqrt{\frac{R}{I}}. \tag{7}$$

If friction is absent, then there is no such restriction on τ.

Let us note in passing that the restriction (7) on the time step obtained here applies not only to the indicated implicit scheme, but also to the explicit Lax scheme described for the flood problem in the book [1]; moreover, in this case it is added to the Courant-Friedrichs-Lewy condition, which relates the steps in time and length.

Condition (7), for large and medium rivers and canals, restricts the time step to quantities of the order of several minutes. Meanwhile, in the propagation of flood waves in rivers, substantial changes in levels and discharges occur over time intervals measured at least in hours. The processes themselves are very prolonged, and therefore calculation with a small step τ requires the expenditure of a large amount of machine time, which in this case is not at all dictated by accuracy requirements, but is caused solely by the requirement of stability of the difference scheme.

In order to get rid of the restriction (7), the difference scheme was slightly modified. The idea of this modification, based on a suggestion by I. M. Gel' fand, consists in taking the quantity Q in the right-hand side of the model equation (5) and, correspondingly, in the friction term entering the right-hand side of equation (2), not at the point k, but at the point $k+1$. Analysis of the model linearized equation, analogous to equation (6), showed that in this case the restriction on the step τ of the form (7) is removed.

To eliminate the nonlinearity in the system of difference equations for z_{k+1} and Q_{k+1}, the quantity Q_{k+1}^2 in them was replaced approximately by the expression

$$Q_{k+1}^2\simeq Q_k^2+2Q_k(Q_{k+1}-Q_k).$$

Practical computations confirmed that this difference scheme is stable for large steps τ.

3. The program compiled for computation on an electronic computer makes it possible to calculate the course of a flood in a system formed by the confluence of two rivers, with each of them, as well as the river below the confluence point, allowed to have a number of concentrated tributary inflows and a tributary inflow distributed along its length. The channel of arbitrary cross section consisted of separate nonprismatic reaches, at the boundaries of which discontinuities in ω, B, and Q could occur (the latter because of the inflow of concentrated tributaries). The matching conditions at such points were formulated on the basis of equality of levels and the discharge balance in them.

A trial calculation was carried out for the propagation of a flood wave along the Zeya River, a large tributary of the Amur. The conditions for the formation of the summer flood of 1956 were taken as the initial data. The calculation was performed for a 620 km reach of the Zeya River (below the town of Zeya) (13 computational reaches) and for its tributary, the Selemdzha, on a 146 km reach before its mouth (3 reaches).

The lateral inflow was determined from the data of hydrometric stations and, where such data were lacking, by the unit hydrograph method. Calculations were also carried out in which the entire inflow was determined by the unit hydrograph method, using data on the course of precipitation over the corresponding parts of the catchment basin. In all, 6 concentrated tributaries were distinguished; the remaining tributary inflow was taken into account as distributed (q). The number of steps in x on each computational reach was taken to be different, on average 8-10 steps. The time step was 1.5 hours.

The course of levels and discharges obtained by calculation was then compared with the results of observations. After some refinement of the initial information, which concerned mainly the sparse data on channel roughness, the discrepancies between calculation and observations at the peaks of the levels were 0.5–1.0 m, with an overall range of level fluctuations of up to 10–12 m.

The motion of the flood wave along the Zeya with the Selemdzha tributary was also calculated for the case in which runoff is regulated by the planned Zeya Hydroelectric Power Station.

The calculations were carried out with the participation of E. V. Shugrina and V. S. Nikiforovskaya.

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Note: Figure translations are in progress. See original paper for figures.