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PHYSICS

A. M. SAMSON, Academician of the Academy of Sciences of the BSSR, B. I. STEPANOV, L. D. KHAZOV

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Abstract

Full Text

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A. M. SAMSON, Academician of the Academy of Sciences of the BSSR, B. I. STEPANOV, L. D. KHAZOV

DEPENDENCE OF THE GENERATION THRESHOLD OF AN OPTICAL GENERATOR ON THE PROPERTIES OF THE RESONATOR

The solution of the nonlinear transport equations for a plane-parallel layer with a negative absorption coefficient leads to the following values of the fluxes of the generated radiation ([1,2]):

$$W_1^{\text{gen}} = \frac{v}{\alpha}\frac{d_1}{\sqrt{r_1}}\frac{-k_0 l - \ln\frac{1}{\sqrt{r_1 r_2}}}{\left(\frac{1}{\sqrt{r_1}}+\frac{1}{\sqrt{r_2}}\right)(1-\sqrt{r_1 r_2})}; \tag{1}$$

$$W_2^{\text{gen}} = \frac{v}{\alpha}\frac{d_2}{\sqrt{r_2}}\frac{-k_0 l - \ln\frac{1}{\sqrt{r_1 r_2}}}{\left(\frac{1}{\sqrt{r_1}}+\frac{1}{\sqrt{r_2}}\right)(1-\sqrt{r_1 r_2})}. \tag{2}$$

Expression (1) corresponds to the flux of radiation emerging normally to the surface of the layer through the boundary with reflection coefficient r_1 and transmittance d_1; expression (2), through the boundary with coefficients r_2 and d_2. The speed of light inside the layer is equal to v, the layer thickness is l, and the nonlinearity parameter is α. The value k_0 corresponds to the absorption coefficient of the substance at low radiation densities. Formulas (1) and (2) were obtained without taking into account scattering of radiation inside the layer. The corresponding expressions for this case are given in ([3]).

The value of the transmittance d in the general case can be represented in the form

$$d = 1 - r - \chi, \tag{3}$$

where χ is a parameter characterizing radiation losses inside the coating.

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Fig. 1

Figure 1: Fig. 1

Expressions (1) and (2) have meaning only when the right-hand side is positive, i.e., when

$$-k_0 > \frac{1}{l} \ln \frac{1}{\sqrt{r_1 r_2}}. \tag{4}$$

Condition (4) is thus the condition for the onset of generation. If it is not satisfied, then stationary luminescence of the layer can exist only in the presence of radiation incident upon it and, in the best case, the system can operate as an amplifier.

Equating the right-hand sides of (1) and (2) to zero,* we obtain the threshold value of the coefficient k_0^{th}

$$k_0^{\text{th}} = -\frac{1}{l} \ln \frac{1}{\sqrt{r_1 r_2}}. \tag{5}$$

The value k_0^{th} thus determines that minimum value of the negative absorption coefficient which is necessary for the onset of the generation regime. The quantity $\frac{1}{l} \ln \frac{1}{\sqrt{r_1 r_2}}$, multiplied by the mean radiation density u, represents the energy losses at the radiation output—

* Expressions (1) and (2) can also be equal to zero as a result of $d_1 = d_2 = 0$ (with r_1 and $r_2 \neq 0$). In this case, generation inside the layer exists when (4) is satisfied, but the radiation is completely absorbed by the coatings.

radiation beyond the layer (calculated per unit length). It is uniquely related to the quality factor of the layer as a resonator. The smaller l, r_1, and r_2 are, the higher the value of k_0^{th}. Conversely, as $r_1 r_2 \to 1$, the value of k_0^{th} also tends to zero, i.e., the generation regime is realized for any negative values of k_0.

Formula (5) is valid in the absence of harmful radiation losses due to the exit of radiation through the lateral faces or its absorption by impurities. If these are taken into account, formula (5) takes the form (see (3)):

$$k_0^{\text{th}} = -\left(\frac{1}{l} \ln \frac{1}{\sqrt{r_1 r_2}} + \sigma\right), \tag{5'}$$

where σ is the fraction of harmful energy losses calculated per unit length.

Fig. 1

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Experimentally, one usually studies the onset of the generation regime as a function of the energy of the external action creating the negative absorption coefficient (the "pumping" power), rather than as a function of the magnitude k_0. Since k_0 is uniquely related to the pump energy, then, knowing k_0^{th}, one can find the threshold value of the energy of the external action. Let us determine the threshold pump energy for a three-level quantum generator. In this case, in accordance with $(4,5)$, the value of the negative absorption coefficient k_{21}^0, corresponding to the transition between the first and second levels (see Fig. 1), is

$$k_{21}^0 = \frac{B_{12}h\nu_{21}n}{v\Delta\nu_{\text{abs}}} \frac{(-d_{12}+d_{21}+A_{21})(p_{31}+p_{32})+P_{31}p_{23}-p_{32}p_{13}}{(A_{21}+d_{21})(p_{32}+p_{13}+p_{31})+d_{12}(p_{32}+p_{23}+p_{31})+p_{13}p_{23}+p_{13}p_{32}+p_{23}p_{31}}. \tag{6}$$

Here ν_{21} is the frequency of the generated radiation; n is the total density of particles; $\Delta\nu_{\text{abs}}$ is the width of the absorption line in the channel $1 \leftrightarrow 2$; A_{ij} and B_{ij} are the integral Einstein coefficients; d_{ij} are the probabilities of nonoptical transitions. The values p_{ij}, defined by the relations

$$\begin{aligned} P_{ij} &= A_{ij}+d_{ij}+B_{ij}u_{ij} \quad (i>j); \\ p_{ji} &= d_{ji}+B_{ij}u_{ij} \quad (i>j), \end{aligned} \tag{7}$$

are equal to the total probabilities of transition from one level to another; u_{ij} is the radiation density of frequency ν_{ij}. Equating (6) to the threshold value

$$-\left(\frac{1}{l}\ln\frac{1}{\sqrt{r_1r_2}}+\sigma\right),$$

and taking (7) into account, we obtain the following expression for the threshold pump power u_{31}^{th}:

$$u_{31}^{\text{th}} = \frac{1}{B_{13}}\frac{L(1+\delta)}{M-N\delta}, \tag{8}$$

where

$$\begin{aligned} L &= (A_{31}+d_{31}+p_{32})(A_{21}+d_{21}-d_{12})+(A_{31}+d_{31})p_{23}-p_{32}d_{13}, \\ M &= p_{32}-p_{23}-A_{21}-d_{21}+d_{12}, \\ N &= 2A_{21}+2d_{21}+2p_{23}+p_{32}+d_{12}, \end{aligned} \tag{9}$$

$$\delta = \frac{v\Delta\nu_{\text{abs}}}{B_{12}h\nu_{21}n}\left(\frac{1}{l}\ln\frac{1}{\sqrt{r_1r_2}}+\sigma\right) = \frac{1}{\varkappa}\left(\frac{1}{l}\ln\frac{1}{\sqrt{r_1r_2}}+\sigma\right). \tag{10}$$

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Here $\varkappa$ is the absorption coefficient in the channel $1 \leftrightarrow 2$ in the absence of pumping (all particles are at level 1).

Formula (8) was obtained within the framework of the probabilistic method and, apparently, has rather broad limits of applicability. On its basis one can study the dependence of the threshold pump power not only on the properties of the resonator and harmful energy losses, but also on the characteristics of the substance, the temperature, particle density, etc.

In a number of cases formula (8) is greatly simplified. Thus, in ruby quantum generators[6] one may approximately assume that $d_{12} = d_{13} \simeq 0$, $A_{21} + d_{21} \ll p_{32}$, $p_{23} \ll p_{32}$. Therefore

$$u_{31}^{\text{thr}} \simeq \frac{A_{21} + d_{21}}{B_{13}} \frac{1+\delta}{1-\delta}. \tag{11}$$

For small values of δ

$$u_{31}^{\text{thr}} = \frac{A_{21} + d_{21}}{B_{13}} (1 + 2\delta). \tag{12}$$

Figure 2 gives graphs of the dependences $B_{31} u_{31}^{\text{thr}}$ on δ for three variants characterized by different values of the transition probabilities. It is seen from the figure that the dependence of u_{31}^{thr} on δ is rectilinear only for small values of δ. As $\delta \to 0$, u_{31}^{thr} tends to a constant value. In the case of applicability of formula (11) or (12), it is equal to

$$\frac{A_{21} + d_{21}}{B_{13}}.$$

This means that, at the threshold pump power, the probability of an induced transition of particles to the third level is approximately equal to the probability of removal of particles from the second level to the first by spontaneous and nonoptical routes. From Fig. 2, and also from formulas (8), (11), (12), it follows that the influence of the parameter δ on the threshold can be very substantial only for sufficiently high δ, i.e., for large losses and small values of x. For small δ, a change in it by several tens of times only slightly shifts the threshold.

Fig. 2. Dependence of the threshold pump power $B_{31} u_{31}^{\text{thr}}$ on δ.
I $-A_{31} + d_{31} = 3 \cdot 10^5$, $p_{32} = 2 \cdot 10^7$, $A_{21} + d_{21} = 0.3 \cdot 10^3$, $d_{12} = d_{13} = p_{23} = 0$ sec^{-1};
II $-A_{31} + d_{31} = 3 \cdot 10^5$, $p_{32} = 2 \cdot 10^7$, $A_{21} + d_{21} = 0.3 \cdot 10^3$, $d_{12} = d_{13} = 0$, $p_{23} = 2 \cdot 10^7$ sec^{-1};
III $-A_{31} + d_{31} = 1.25 \cdot 10^8$, $p_{32} = 1.25 \cdot 10^9$, $A_{21} + d_{21} = 4.5 \cdot 10^2$, $d_{12} = d_{13} = p_{23} = 0$ sec^{-1}.

Table 1

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x	r_1	r_2	σ	δ	$u_{31}^{\text{thr}}\dfrac{B_{13}}{A_{21}+d_{21}}$
0.4	0.99	0.99	0	0.004	1.008
4	0.99	0.99	0	0.0004	1.0008
16	0.99	0.99	0	0.0001	1.0002
0.4	0.99	0.85	0	0.036	1.072
4	0.99	0.85	0	0.004	1.008
16	0.99	0.85	0	0.0009	1.002
0.4	0.99	0.70	0.1	0.326	1.965
4	0.99	0.70	0.1	0.033	1.067
16	0.99	0.70	0.1	0.008	1.017
0.4	0.99	0.99	0.3	0.75	7
4	0.99	0.99	0.3	0.075	1.16
16	0.99	0.99	0.3	0.020	1.04

As follows from (10), changes in δ can be achieved in various ways: by changing the width of the absorption band $\Delta\nu_{\text{abs}}$ in the channel $1 \leftrightarrow 2$, the density of atoms of the active substance n, the Einstein coefficient B_{12}, the velocity of light v, the quality of the plane-parallel layer as a resonator, and the magnitude of the harmful losses σ. Since Δ_{abs} and the probabilities of many transitions (for example, p_{23}) depend on temperature, formulas (8), (11), (12) can serve as a basis for studying the temperature dependence of the threshold pump power u_{31}^{thr}.

According to (11), the value of u_{31}^{thr} is positive for $\delta < 1$ (or, according to (8), for $N\delta < M$). For

$$\frac{1}{l}\ln\frac{1}{\sqrt{r_1 r_2}} + \sigma > x \tag{13}$$

generation is impossible altogether, since even at very high pumpings the radiation losses at the frequency ν_{21} cannot be compensated by its amplification in stimulated emission.

It should be noted that, for a good resonator quality (high reflection coefficients r_1 and r_2, large layer thickness l, and small harmful losses σ), the value of δ is always small. In this case, a strong change in the number of particles n, the width $\Delta\nu_{\text{abs}}$, etc., has practically no effect on the threshold value.

A reduction of the threshold pump power by increasing n or decreasing $\Delta\nu_{\text{abs}}$ (with lowering of the temperature) can be achieved only in poor resonators (small r_1, r_2, l, large σ). In Table 1, for illustration, values of δ and

$$u_{31}^{\text{thr}}\frac{B_{13}}{A_{21}+d_{21}}$$

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are calculated for several values of χ, r_1, r_2, σ characteristic of ruby. The value of l was taken to be 6 cm.

Institute of Physics
Academy of Sciences of the BSSR

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