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Abstract

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HYDRODYNAMIC FORECASTING OF PRESSURE FIELDS AT THE MIDDLE LEVEL OF THE ATMOSPHERE FOR THE ENTIRE SPHERE OF THE EARTH, TAKING OROGRAPHY INTO ACCOUNT

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The problem of forecasting, with short and long lead times, the pressure field at the middle level of the troposphere for the Northern Hemisphere of the Earth is at present well developed ([1,2]). The problem of forecasting for the entire terrestrial globe has been studied much less: the need for forecasting in the equatorial zones greatly complicates the problem.

In forecasting for the entire terrestrial globe we shall start from the nonlinear vorticity-transport equation for the middle level and the linearized balance equation ([3]):

$$\Delta\frac{\partial\psi}{\partial\theta}+\frac{1}{a_0^2\sin\theta}(\psi,\ \Delta\psi+2\omega a_0^2 h)+2\omega\frac{\partial\psi}{\partial\lambda}=0, \tag{1}$$

$$\cos\theta\Delta\psi-\sin\theta\frac{\partial\psi}{\partial\theta}=\frac{g}{2(\alpha+\omega)}\Delta H, \tag{2}$$

written in the spherical coordinate system θ (complement of latitude), λ (longitude of the point). Here ψ is the stream function; H is the height of the 500 mb isobaric surface; α is the circulation index; $\tilde{h}=\varepsilon\cos\theta\, gh(\theta,\lambda)/RT_1$, where $h(\theta,\lambda)$ is the height of the Earth' s surface above sea level; T_1 is the mean temperature of the entire atmosphere; ε is a smoothing factor ($\varepsilon<1$). The remaining notation is from ([4]).

The problem is divided into three parts: 1 —determining the field ψ for the initial time from the known initial distribution of the height H; 2 —forecasting ψ for any time; 3 —determining H from the predicted values of ψ.

Problem 1. We take the height of the 500 mb surface at the nodes of a degree grid with intervals in λ of 10° and in θ of 5°. We represent H on each circle of latitude by an interpolation trigonometric polynomial

$$H(\theta, \lambda, 0) = \frac{\tilde{a}_0(\theta)}{2} + \sum_{m=1}^{18} \left[a_m(\theta) \cos m\lambda + b_m(\theta) \sin m\lambda \right]; \tag{3}$$

we shall also seek ψ in the form of a trigonometric polynomial

$$\psi(\theta, \lambda, 0) = \frac{c_0(\theta)}{2} + \sum_{m=1}^{18} \left[c_m(\theta) \cos m\lambda + d_m(\theta) \sin m\lambda \right]. \tag{4}$$

Substitute the expressions for H and φ from (3) and (4) into equation (2), replace the derivatives with respect to λ by finite differences, and compare coefficients–we obtain ordinary second-order differential equations in θ, relating the coefficients of the polynomial for H to the coefficients of the polynomial for ψ.

Fig. 1

a

b

Fig. 2

a

b

It can be shown that the zonal components $\frac{1}{2}\bar{c}_0(\theta) = c_0(\theta)$ are determined through $\frac{1}{2}\bar{a}_0(\theta) = a_0(\theta)$ from the ordinary first-order differential equation

$$\cos\theta \, dc_0(\theta)/d\theta = \gamma \, da_0(\theta)/d\theta, \tag{5}$$

where $\gamma = g/2(\alpha + \omega)$; at the same time it should be remembered that the stream function is determined only up to a constant.

The coefficients $c_m(\theta)$ are determined from the equation

$$\frac{d}{d\theta}\left(\frac{1}{2}\sin 2\theta \frac{dc_m(\theta)}{d\theta}\right) - \lambda_m^2(\theta) \operatorname{ctg}\theta \, c_m(\theta) =$$

$$= \gamma \left[\frac{d}{d\theta}\left(\sin\theta \frac{da_m(\theta)}{d\theta}\right) - \frac{\lambda_m^2(\theta)}{\sin\theta} a_m(\theta) \right], \tag{6}$$

where $\lambda_m(\theta) = m \sin\left[m(\Delta\lambda)_\theta\right]/m(\Delta\lambda)_\theta$ $(2(\Delta\lambda)_\theta = 4.5\,\Delta\lambda$ for $\theta = 5^\circ, \theta = 175^\circ$; $2.65\,\Delta\lambda$ for $\theta = 10^\circ, \theta = 170^\circ$; $1.85\,\Delta\lambda$ for $\theta = 15^\circ, \theta = 165^\circ$; $\Delta\lambda/2\sin\theta$ for $20^\circ \leq \theta \leq 160^\circ$; $\Delta\lambda = \pi/18)$. From an analogous equation the coefficients $d_m(\theta)$ are determined through $b_m(\theta)$.

In solving equation (6), difficulties arise that are associated with the coefficient at the highest derivative becoming zero at the equator ($\theta = \pi/2$). To avoid this difficulty, we introduce two functions

$$a_m(\theta)_1 = a_m(\theta) - a_m(\pi - \theta), \qquad a_m(\theta)_2 = a_m(\theta) + a_m(\pi - \theta).$$

It is obvious that $a_m(\theta)_1$ and $a_m(\theta)_2$ are functions, respectively, antisymmetric and symmetric with respect to the equator. We also introduce the functions $c_m(\theta)_1$ and $c_m(\theta)_2$, which are the solution of equation (6) for $a_m(\theta)$, respectively equal to $a_m(\theta)_1$ and $a_m(\theta)_2$. From equation (6) it is seen that $c_m(\theta)_1$ is a function symmetric with respect to the equator, while $c_m(\theta)_2$ is antisymmetric. By constructing these functions we reduce the interval in θ of our problem to $0 < \theta \leq \pi/2$. Knowing $c_m(\theta)_1$ and $c_m(\theta)_2$ on this interval, we find c_m from the formulas

$$c_m(\theta) = \frac{1}{2}\left[c_m(\theta)_1 + c_m(\theta)_2\right], \qquad c_m(\pi - \theta) = \frac{1}{2}\left[c_m(\theta)_1 - c_m(\theta)_2\right].$$

Introducing the notation $\Gamma_m(\theta)_\nu = dc_m(\theta)_\nu/d\theta$, $V_m(\theta)_\nu = da_m(\theta)_\nu/d\theta$ and replacing derivatives by finite differences, we arrive at the following system of algebraic equations containing the unknowns $c_m(\theta), \Gamma_m(\theta)_\nu$:

$$c_{m\nu}^{i+1} - c_{m\nu}^{i} = \frac{1}{2}\Delta\theta\left(\Gamma_{m\nu}^{i+1} + \Gamma_{m\nu}^{i}\right),$$

$$\sin\theta_i\left[\sin\theta_{i+1}\left(\cos\theta_{i+1}\Gamma_{m\nu}^{i+1} - \gamma V_{m\nu}^{i+1}\right) - \sin\theta_i\left(\cos\theta_i\Gamma_{m\nu}^{i} - \gamma V_{m\nu}^{i}\right)\right]$$

$$= \frac{\Delta\theta}{2}\left[\sin\theta_i\frac{\left(\lambda_m^{i+1}\right)^2}{\sin\theta_{i+1}}\left(\cos\theta_{i+1}c_{m\nu}^{i+1} - \gamma a_{m\nu}^{i+1}\right) + \left(\lambda_m^{i}\right)^2\left(\cos\theta_i c_{m\nu}^{i} - \gamma a_{m\nu}^{i}\right)\right] \qquad (7)$$

$$(i = 1, 2, \ldots, 17;\ \theta_i = i\Delta\theta;\ \Delta\theta = \pi/36;\ \nu = 1, 2)$$

under the conditions: $c_m(0)_1 = c_m(0)_2 = 0$, $\Gamma_m(\pi/2)_1 = 0$, $c_m(\pi/2)_2 = 0$ (with $a_m(0)_1 = a_m(0)_2 = 0$, $a_m(\pi/2)_1 = 0$, $V_m(\pi/2)_2 = 0$). To system (7) one more equation must be appended. We shall obtain it if we write equation (6) in finite differences in the form

$$\sin 2\theta_{j-1/2}c_{m\nu}^{j-1} - 2\left[\sin 2\theta_j\cos 5^\circ + \left(\lambda_m^j\right)^2\operatorname{ctg}\theta_j\right]c_{m\nu}^{j} + \sin 2\theta_{j+1/2}c_{m\nu}^{j+1}$$

$$= 2\gamma \left[\sin\theta_{j-1/2} a_{m\nu}^{j-1} - \left(2\sin\theta_j \cos 2.5^\circ + \frac{\left(\lambda_m^j\right)^2 (\Delta\theta)^2}{\sin\theta_j} a_{m\nu}^i \right) + \sin\theta_{j+1/2} a_{m\nu}^{j+1} \right]$$

$$(\nu = 1, 2) \tag{8}$$

and put $j = 1$.

Let us determine from each pair of equations of system (7) $\Gamma_{m\nu}^i, \Gamma_{m\nu}^{i+1}$ through $c_{m\nu}$ and $a_{m\nu}$ with the corresponding upper values; taking in $\Gamma_{m\nu}^i$ $i = j+1$, and in $\Gamma_{m\nu}^{i+1}$ $i = j$, and equating the expressions thus obtained for

Γ_{mv}^{i+1}, we arrive at a system of linear algebraic equations only with respect to the unknowns c_{mv}. This system of equations, together with equation (8), written for $j = 1$, is solved by the sweep method.

Problem 2. We seek the solution in the form of the polynomial

$$\psi(\theta, \lambda, t) = \sum_{m=0}^{18} \left[c_m(\theta, t) \cos m\lambda + d_m(\theta, t) \sin m\lambda \right]; \tag{9}$$

$\Delta\psi(\theta, \lambda, t)$ is represented by an analogous polynomial with coefficients $\Omega_m(\theta, t)$, $\Omega'_m(\theta, t)$.

We also represent $\tilde{h}$ in the form of a polynomial with coefficients $h_m(\theta)$, $h'_m(\theta)$. Let

$$\Delta\psi + 2\omega a_0^2 \tilde{h} = \sum_{m=0}^{18} \left[\omega_m(\theta, t) \cos m\lambda + \omega'_m(\theta, t) \sin m\lambda \right]. \tag{10}$$

Substituting (9) and (10) into (1) and comparing coefficients, we obtain

$$\frac{\partial \Omega_m}{\partial t} = -\frac{1}{a_0^2 \sin\theta} \left(\tilde{S}_m - \widetilde{\widetilde{S}}_m \right) - 2\omega \overline{\lambda}_m d_m,$$

$$\frac{\partial \Omega'_m}{\partial t} = -\frac{1}{a_0^2 \sin\theta} \left(\tilde{S}'_m - \widetilde{\widetilde{S}}'_m \right) + 2\omega \overline{\lambda}_m c_m \tag{11}$$

$$(m = 0, 1, 2, \dots, 18;\ \theta = 5, 10, \dots, 175^\circ),$$

where

$$\overline{\lambda}_k = \lambda_k [(\Delta\lambda)_\theta],$$

$$\tilde{S}_0 = \sum_{k=1}^{18} \overline{\lambda}_k \left(\frac{\partial c_k}{\partial \theta} \omega'_k - \frac{\partial d_k}{\partial \theta} \omega_k \right),$$

$$\tilde{S}_m = \frac{1}{2} \left[\sum_{k=1}^{m} \overline{\lambda}_k \left(\omega'_k \frac{\partial c_{m-k}}{\partial \theta} + \omega_k \frac{\partial d_{m-k}}{\partial \theta} \right) + \right.$$

$$\left. + \sum_{k=1}^{18-k} \overline{\lambda}_k \left(\omega'_k \frac{\partial c_{m+k}}{\partial \theta} - \omega_k \frac{\partial d_{m+k}}{\partial \theta} \right) + \sum_{k=m}^{18} \overline{\lambda}_k \left(\omega'_k \frac{\partial c_{k-m}}{\partial \theta} - \omega_k \frac{\partial d_{k-m}}{\partial \theta} \right) \right],$$

$$\tilde{S}'_m = \frac{1}{2} \left[\sum_{k=1}^{m} \overline{\lambda}_k \left(\omega'_k \frac{\partial d_{m-k}}{\partial \theta} - \omega_k \frac{\partial c_{m-k}}{\partial \theta} \right) + \right.$$

$$\left. + \sum_{k=1}^{18-m} \overline{\lambda}_k \left(\omega'_k \frac{\partial d_{m+k}}{\partial \theta} + \omega_k \frac{\partial c_{m+k}}{\partial \theta} \right) - \sum_{k=m}^{18} \overline{\lambda}_k \left(\omega_k \frac{\partial c_{k-m}}{\partial \theta} + \omega'_k \frac{\partial d_{k-m}}{\partial \theta} \right) \right].$$

$\widetilde{\widetilde{S}}_0$ is obtained from $\tilde{S}_0$, $\widetilde{\widetilde{S}}_m$ from $\tilde{S}_m$, and $\acute{\widetilde{\widetilde{S}}}_m$ from $\tilde{S}'_m$ by the formal replacement of c_j by ω_j, ω_j by c_j, d_j by ω'_j, and ω'_j by d_j, for any lower index. For the poles we obtain:

$$\frac{\partial \Omega_0^0}{\partial t} = \frac{1}{a_0^2 (\Delta\theta)^2} \sum_{\rho=1}^{9} \sum_{s=1}^{9} (-1)^{s+1} \left(d_{2s-1}^1 \omega'^{1}_{2\rho-1} - c_{2\rho-1}^1 \omega_{2s-1}^1 \right),$$

$$\frac{\partial \Omega_0^{36}}{\partial t} = \frac{1}{a_0^2 (\Delta\theta)^2} \sum_{\rho=1}^{9} \sum_{s=1}^{9} (-1)^{s+1} \left(c_{2\rho-1}^{35} \omega_{2s-1}^{35} - d_{2s-1}^{35} \omega_{2\rho-1}^{35} \right). \tag{12}$$

When multiplying the series in the expression for the Jacobian from (1), we discard all terms containing $k > 18$.

From the initial values c_m and d_m and from the expansion coefficients $\tilde{h}$, the initial ω_m, ω'_m and $\widetilde{S}_0$, $\widetilde{S}_m, \dots, \widetilde{S}'_m$ are determined. After this the right-hand sides of (11), (12) are computed and one time step is taken for Ω_m, Ω'_m. From the future values Ω_m, Ω'_m we determine the future c_m, d_m, ω_m, ω'_m. We can now proceed to the next time step.

Problem 3. Since the differential equation (6) has no singularities with respect to a_m, a_m can be determined from the system (8) for $j = 1, \dots, 35$, with the pole conditions $c_m^0 = a_m^0 = c_m^{36} = a_m^{36} = 0$. The zonal components are determined from equation (5) with the closing condition

$$\int_0^{2\pi}\int_0^{\pi} H(\theta,\lambda,t)\sin\theta\,d\theta\,d\lambda = \int_0^{2\pi}\int_0^{\pi} H(\theta,\lambda,0)\sin\theta\,d\theta\,d\lambda, \tag{13}$$

which is equivalent to satisfying the condition

$$\int_0^{2\pi}\int_0^{\pi} \frac{\partial H(\theta,\lambda,t)}{\partial t}\sin\theta\,d\theta\,d\lambda = 0.$$

We give an example of a forecast for 1 day ahead for the entire globe. As the initial data, the data for 03:00 Moscow time on 19 VI 1958 were used (Fig. 1); the time step was taken equal to 3 hours. The forecast is given in Fig. 2. The actual charts for the forecast time are given in Fig. 3.

In conclusion, I consider it my duty to express my gratitude to Corresponding Member of the Academy of Sciences of the USSR E. N. Blinova for posing the problem and for valuable advice.

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Note: Figure translations are in progress. See original paper for figures.