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Abstract

Full Text

GEOPHYSICS

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A SPATIAL PROBLEM ON THE MOTION OF A COLD LAYER OF AIR OVER MOUNTAINS SMALL IN COMPARISON WITH THE THICKNESS OF THE LAYER

(Presented by Academician A. A. Dorodnitsyn on 5 VI 1962)

The linear spatial problem of the flow of an air current around mountains was first solved most completely by A. A. Dorodnitsyn (¹). The solution obtained in (¹) is expressed in the form of very complicated integrals. In the work (²), I. A. Kibel constructed a linear equation for the vertical velocity over mountain slopes and obtained its solution, assuming that the air flow is bounded above by a fixed horizontal plane. In the works (¹, ²), the Coriolis force was not taken into account. Sh. A. Musaelyan considered (³) the linear spatial problem of the flow around large mountain massifs, taking into account the sphericity of the Earth and the Coriolis force.

In the present paper a solution is given of the linearized spatial problem of the motion of a cold layer of air over rugged relief, on the basis of the nonlinear problem posed by F. I. Frankl and L. N. Gutman (⁴). The proposed work differs from the above-mentioned linear problems by the comparative simplicity of the mathematical apparatus and by the presence, at the upper boundary of the flow, of a free surface, whose form is determined as a result of the solution with allowance for the Coriolis force.

The authors of (⁴) represent the atmosphere in the form of two layers: an upper one, in which the motion is assumed to be quasigeostrophic and prescribed, and a lower one, whose motion is to be found. The air temperature in both layers decreases linearly with height, but at the interface of the layers it has a positive jump $\Delta\theta = \text{const} > 0$. The quantity $\Delta\theta$ characterizes the thermal difference of the layers.

Let the relief, described by the equation $z = \delta(x, y)$, as $\sqrt{x^2 + y^2} \rightarrow \infty$ pass into a horizontal plain, over which the thickness of the lower layer $h(x, y)$, the velocity components $u(x, y)$, $v(x, y)$, and the stream function $\psi(x, y)$ take the form:

$$h = H = \text{const}; \quad u = U = \text{const}; \quad v = 0; \quad \psi = UHy. \quad (1)$$

Taking into account the conditions (1), we write the system of equations obtained in the work (⁴) in the form

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = l \left[1 - (1 - \sigma\delta) \frac{h}{H} \right] \quad \left(\sigma = \frac{\partial \ln \rho}{\partial z} \simeq \text{const} \right); \quad (2)$$

$$\frac{1}{2} (u^2 + v^2 - U^2) + \mu\xi + \nu\xi^2 + \frac{l}{H}\psi - lUy = 0$$

$$\left(\mu = \frac{g}{\theta_H} \Delta\theta; \quad \nu = \frac{g}{\theta_H} \frac{\gamma_a - \gamma_T}{2} \right); \quad (3)$$

$$u = \frac{1}{(1 - \sigma\delta)h} \frac{\partial \psi}{\partial y}; \quad v = -\frac{1}{(1 - \sigma\delta)h} \frac{\partial \psi}{\partial x}; \quad (4)$$

$$h + \delta = H + \xi. \quad (5)$$

Here $l \approx \text{const}$ is the Coriolis parameter; g is the acceleration due to gravity; θ_H is the absolute temperature in the upper layer at the level H ; γ_a is the adiabatic vertical temperature gradient; γ_m is the vertical temperature gradient in the upper layer; ρ is the air density; ζ is the deviation of the interface of the layers from the level H .

Let us consider mountain masses whose height $\delta \ll H$. The latter condition makes natural the assumption that comparatively low mountains (shallow valleys) produce small perturbations

$$u = U + u', \quad v = v', \quad h = H + \zeta - \delta, \quad \psi = UHy + \psi', \quad (6)$$

where $u', v', \zeta, \delta, \psi'$ are quantities of the first order of smallness in comparison with U, H, UHy .

Substituting relations (6) into (2)–(5), neglecting quantities of the second order of smallness, as well as the very small quantity $\sigma\delta$, we obtain the equation for ψ' :

$$(1 - M^2) \left[\frac{\partial^2 \psi'}{\partial x^2} + \frac{\partial^2 \psi'}{\partial y^2} - k^2 \psi' \right] = -U \frac{\partial \delta}{\partial y} + l\delta; \quad (7)$$

where $M = U/a$; $k = l/a$; $a = \sqrt{\mu H}$, by analogy with the theory of shallow water, will be called the velocity of surface waves. For $M < 1$ equation (7) is of elliptic type, and for $M > 1$ it is of hyperbolic type.

Let us find the solution of equation (7) for $M < 1$.

The boundary condition for equation (7) has the form

$$\psi' = 0 \quad \text{as} \quad \sqrt{x^2 + y^2} \rightarrow \infty. \quad (8)$$

We introduce dimensionless functions and independent variables by the formulas

$$\delta = \delta_m \bar{\delta}; \quad x = L\sqrt{1 - M^2} \bar{x}; \quad y = L\bar{y}; \quad \psi' = UL\delta_m \bar{\psi},$$

where δ_m is the maximum height of the mountains; L is the characteristic horizontal scale.

The problem (7), (8) in dimensionless form takes the form (bars over dimensionless quantities are omitted):

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - \lambda^2 \psi = -\frac{\partial \delta}{\partial y} + \frac{\lambda}{M} \delta \quad \left(\lambda = \frac{LL}{a} \right); \quad (9)$$

$$\psi = 0 \quad \text{as} \quad \sqrt{x^2 + y^2} \rightarrow \infty. \quad (10)$$

Applying Green's method to the solution of problem (9), (10), we obtain for the function ψ the integral (5)

$$\psi(x, y) = -\frac{1}{2\pi} \iint_{-\infty}^{\infty} \left[-\frac{\partial \delta(\xi, \eta)}{\partial \eta} + \frac{\lambda}{M} \delta(\xi, \eta) \right] K_0 \left[\lambda \sqrt{(x - \xi)^2 + (y - \eta)^2} \right] d\xi d\eta, \quad (11)$$

where K_0 is the Macdonald function of order zero.

On the basis of (3)–(5), the quantities u', v', ζ are expressed in terms of the dimensionless functions ψ, δ as follows:

$$u' = \frac{1}{1 - M^2} \frac{\delta_m}{H} \left(\frac{\partial \psi}{\partial y} + \lambda M \psi + \delta \right) U; \quad (12)$$

$$v' = -\frac{\delta_m}{H} \frac{1}{\sqrt{1 - M^2}} \frac{\partial \psi}{\partial x} U; \quad (13)$$

$$\zeta = -\frac{M^2}{1 - M^2} \left(\frac{\partial \psi}{\partial y} + \frac{\lambda}{M} \psi + \delta \right) \delta_m. \quad (14)$$

Expression (11) shows that the perturbations decay monotonically, since K_0 is a monotonically decreasing function.

Let us write $\psi(x, y)$ in the form $\psi = \psi_1 + \psi_2$, where

Fig. 1

Figure 1: Fig. 1

$$\psi_1 = \frac{1}{2\pi} \iint_{-\infty}^{\infty} \frac{\partial \delta(\xi, \eta)}{\partial \eta} K_0 \left[\lambda \sqrt{(x - \xi)^2 + (y - \eta)^2} \right] d\xi d\eta; \quad (15)$$

$$\psi_2 = -\frac{\lambda}{2\pi M} \iint_{-\infty}^{\infty} \delta(\xi, \eta) K_0 \left[\lambda \sqrt{(x - \xi)^2 + (y - \eta)^2} \right] d\xi d\eta. \quad (16)$$

We shall assume that $z = \delta(x, y)$ and its derivatives up to and including the second order are continuous, and also that δ is symmetric with respect to the plane xz . Integrals (15) and (16) show that the magnitude of the perturbations is affected by two principal factors: the steepness of the mountain's side slopes and the height of the mountain. The contribution of the steepness of the side slopes according to (15) reduces to a distortion of the streamlines that is symmetric with respect to the x -axis. In the case of a valley the streamlines become denser toward its longitudinal axis, while in the case of a mountain the streamlines move away from the axis of symmetry. Integral (16), which arises as a result of taking into account the rotation of the Earth, is positive for $\delta < 0$ (valleys) and negative for $\delta > 0$ (mountains). The presence of ψ_2 leads to a condensation of the streamlines on the left (relative to the general direction of motion) slope of the mountain. In the case of valleys the streamlines are pressed to the right.

Fig. 1

If the ratio λ/M is small (low hills), then integral (15) plays an essential role. Over the mountain we obtain an increase of the wind, and the boundary between the cold and warm layers bends downward.

For large λ/M (plateaus of horizontal scale of several hundred kilometers), the principal importance is acquired by integral (16), which leads to the wind speed on the left slope of the plateau being greater than on the right.

For $\lambda/M \sim 1$, both integrals (15) and (16) are important.

Figure 1 shows the streamlines for flow around an elevation described by the dimensionless equation $\delta = -e^{-3[x^2(1-M^2)+y^2]}$ for the parameter values $\lambda = 4$, $M = \delta_m/H = 0.25$. It is seen that the Coriolis force plays an important role in flow around large elevations.

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Note: Figure translations are in progress. See original paper for figures.

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