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1962

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Abstract

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GREEN FUNCTIONS IN THE STRONG-COUPLING THEORY

(Presented by Academician N. N. Bogolyubov on 3 V 1962)

1. The strong interaction of an extended nucleon with a charged meson field has been studied on models in a number of works ([1], [2]), all the authors attempting in one way or another to diagonalize the Hamiltonian of the system and thereby determine both the ground-state energy and the renormalized meson charge. In doing so, attempts to pass to a point nucleon (removal of the cutoff) led to a logarithmic divergence of the renormalized coupling constant ([2]). There is reason to suppose that this is connected with the presence in the system of degeneracy with respect to rotations in isotopic space. Therefore, in studying the interaction of a point nucleon with a charged meson field we shall remove this degeneracy, correspondingly changing the Hamiltonian of the system. In contrast to works ([1], [2]), we shall use a more convenient method connected with the application of Green functions ([3], [4]).

2. Let us take the Hamiltonian of a system of charged mesons interacting with an infinitely heavy nucleon in the form:

$$\mathcal{H} = \sum_{(k)} \omega_k \left(b_{k+}^+ b_{k+} + b_{k-}^+ b_{k-} \right) - g \left(Q\tau + \tau^+ Q^+ \right) + v \left(Q - Q^+ \right)^2, \qquad (1)$$

where

$$Q = \sum_{(k)} \frac{\lambda_k}{\sqrt{2\omega_k}} \left(b_{k+} + b_{k-}^+ \right); \qquad Q^+ = \sum_{(k)} \frac{\lambda_k}{\sqrt{2\omega_k}} \left(b_{k-} + b_{k+}^+ \right); \qquad (2)$$

τ, τ^+ are the creation and annihilation operators of the nucleon charge, possessing the property $\tau\tau^+ + \tau^+\tau = 1$; $b_{k+}^+ (b_{k+})$ and $b_{k-}^+ (b_{k-})$ are the creation (annihilation) operators of positive and negative mesons with momentum k; $\lambda_k = \lambda_k^* = \lambda_{(k^2)}$ is a form factor equal to 1 for a point nucleon, and $v \neq 0$ is a real number.

Owing to the addition to the ordinary Hamiltonian of the term $v(Q - Q^+)^2$ ([3]), the total charge of the system

$$q = \tau\tau^+ + \sum_{(k)} b_{k+}^+ b_{k+} - \sum_{(k)} b_{k-}^+ b_{k-} \tag{3}$$

ceases to be an integral of motion, which leads to the removal of the rotational degeneracy in isotopic space. Indeed, if $v \neq 0$, Hamiltonian (1) is not invariant with respect to the simultaneous replacement of the operators:

$$b_{k+} \to e^{i\varphi} b_{k+}; \qquad b_{k-} \to e^{-i\varphi} b_{k-}; \qquad \tau \to \tau e^{-i\varphi}; \tag{4}$$

$$b_{k+}^+ \to e^{-i\varphi} b_{k+}^+; \qquad b_{k-}^+ \to e^{i\varphi} b_{k-}^+; \qquad \tau^+ \to \tau^+ e^{i\varphi},$$

where φ is a real phase. The removal of charge degeneracy makes it possible to obtain a finite renormalization of the meson charge not only for an extended, but also for a point nucleon.

3. Following the work ([4]), let us construct equations for the retarded Green functions of commutator type. Taking into account the equations of motion for the Heisenberg operators τ, τ^+ and $\sigma = \tau\tau^+ - \tau^+\tau$, we obtain, in the energy representation:

$$\begin{aligned} E\langle\langle \tau \mid \tau^+ \rangle\rangle &= \frac{\langle\sigma\rangle}{2\pi} - g\langle\langle \sigma Q^+ \mid \tau^+ \rangle\rangle; \\ E\langle\langle \tau^+ \mid \tau^+ \rangle\rangle &= g\langle\langle \sigma Q \mid \tau^+ \rangle\rangle; \\ E\langle\langle \sigma \mid \tau^+ \rangle\rangle &= -\frac{\langle\tau^+\rangle}{\pi} - 2g\langle\langle \tau Q \mid \tau^+ \rangle\rangle + 2g\langle\langle \tau^+ Q^+ \mid \tau^+ \rangle\rangle. \end{aligned} \tag{5}$$

Here the averaging is carried out over the ground state of the Hamiltonian (1).

To solve this system of equations, by analogy with Wick' s theorem, we decouple the higher Green functions, assuming that in the principal approximation $\langle\tau\rangle = \langle\tau^+\rangle$, $\langle\sigma\rangle = 0$.

Then we obtain:

$$\begin{aligned} E\langle\langle \tau \mid \tau^+ \rangle\rangle &= -g\langle Q^+\rangle\langle\langle \sigma \mid \tau^+ \rangle\rangle; \\ E\langle\langle \tau^+ \mid \tau^+ \rangle\rangle &= g\langle Q\rangle\langle\langle \sigma \mid \tau^+ \rangle\rangle; \\ E\langle\langle \sigma \mid \tau^+ \rangle\rangle &= -\frac{\langle\tau\rangle}{\pi} - 2g\langle\tau\rangle\langle\langle Q \mid \tau^+ \rangle\rangle - 2g\langle Q\rangle\langle\langle \tau \mid \tau^+ \rangle\rangle \\ &\quad + 2g\langle\tau\rangle\langle\langle Q^+ \mid \tau^+ \rangle\rangle + 2g\langle Q^+\rangle\langle\langle \tau^+ \mid \tau^+ \rangle\rangle. \end{aligned} \tag{6}$$

The mean values $\langle Q\rangle$ and $\langle Q^+\rangle$ are determined from the equations of motion for the meson operators:

$$\langle Q\rangle = \langle Q^+\rangle = gI\langle\tau\rangle, \tag{7}$$

where

$$I = \sum_{(\mathbf{k})} \frac{\lambda_k^2}{\omega_k^2}, \tag{8}$$

and the Green functions of mixed type are expressed through the nucleon Green functions in the following way:

$$\langle\langle Q \mid \tau^+\rangle\rangle = \frac{gJ(E)}{1-4\nu J(E)} \left\{(1-2\nu J(E))\langle\langle\tau^+ \mid \tau^+\rangle\rangle - 2\nu J(E)\langle\langle\tau \mid \tau^+\rangle\rangle\right\}; \tag{9}$$

$$\langle\langle Q^+ \mid \tau^+\rangle\rangle = \frac{gJ(E)}{1-4\nu J(E)} \left\{-2\nu J(E)\langle\langle\tau^+ \mid \tau^+\rangle\rangle + (1-2\nu J(E))\langle\langle\tau \mid \tau^+\rangle\rangle\right\},$$

where

$$J(E) = \sum_{(\mathbf{k})} \frac{\lambda_k^2}{\omega_k^2 - E^2}. \tag{10}$$

Then, from the system of equations (6), we obtain:

$$\begin{aligned}
\langle\langle\tau \mid \tau^+\rangle\rangle &= \frac{1}{\pi} \frac{g^2 I \langle\tau\rangle^2}{E^2 - \dfrac{4g^4\langle\tau\rangle^2 I^2}{1-4\nu J(E)}\Delta(E)}; \\
\langle\langle\tau^+ \mid \tau^+\rangle\rangle &= -\langle\langle\tau \mid \tau^+\rangle\rangle; \\
\langle\langle\sigma \mid \tau^+\rangle\rangle &= -\frac{E}{\pi} \frac{\langle\tau\rangle}{E^2 - \dfrac{4g^4\langle\tau\rangle^2 I^2}{1-4\nu J(E)}\Delta(E)},
\end{aligned} \tag{11}$$

where

$$\Delta(E) = 1 - 4\left(\nu + \frac{1}{4I}\right) J(E). \tag{12}$$

4. A similar result can also be obtained by means of perturbation theory in inverse powers of the interaction constant g. Indeed, representing the solution of the Schrödinger equation

$$\mathcal{H}\Psi = E\Psi \tag{13}$$

in the form of an expansion in the eigenfunctions of the nucleon charge operators τ and τ^+:

$$\Psi = \Phi_p \delta(n_p - 1)\delta(n_n) + \Phi_n \delta(n_n - 1)\delta(n_p), \tag{14}$$

where Φ_p and Φ_n depend only on the meson operators, we obtain, in the zeroth approximation in powers of g^{-1}, the following equation for the ground state $A_0 = \Phi_p + \Phi_n$:

$$(\mathcal{H}^0 - E^0)A_0 = 0. \tag{15}$$

The Hamiltonian

$$\mathcal{H}^0 = \sum_{(\mathbf{k})} \omega k \left(b^+_{k+} b_{k+} + b^+_{k-} b_{k-}\right) + \left(v + \frac{1}{4I}\right)(Q - Q^+)^2 \tag{16}$$

can be diagonalized by means of the u, v-transformation ([5]), and its eigenvalues E^0_μ are determined as the roots of the secular equation

$$\Delta(E^0) = 0. \tag{17}$$

It is easy to see that all $E^0_\mu \neq 0$.
Since $\Delta(E)$ also appears in the denominators of the nucleon Green functions, all three Green functions (11) have no poles at $E = 0$, which indicates the absence of any kinds of degeneracies in the system.

5. The Green functions written out make it possible to determine the mean values of the operators for a point nucleon.

In the principal approximation we obtain:

$$\langle\tau\rangle = \langle\tau^+\rangle = \frac{1}{2}. \tag{18}$$

In contrast to the degenerate case ([1,2]), all mean values are finite. Defining the renormalized coupling constant as $g_r = g\langle\tau\rangle$, we have:

$$g_r = \frac{1}{2} g. \tag{19}$$

The same result can also be obtained by means of perturbation theory in powers of g^{-1}.

It is interesting to note that, in weak coupling as well, removal of the degeneracy leads to a finite renormalization of the charge g for a point nucleon.

The author expresses his deep gratitude to Acad. N. N. Bogolyubov for constant attention and valuable advice, and to V. D. Kukin for useful discussions.

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Received
28 IV 1962

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Note: Figure translations are in progress. See original paper for figures.