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Abstract

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ESTIMATION OF THE VERTICAL DIFFUSION COEFFICIENT OF A SETTLING ADMIXTURE IN THE ATMOSPHERE FROM ITS DISTRIBUTION ON THE EARTH'S SURFACE

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Estimation of the diffusion coefficients of an admixture settling in the atmosphere is a very laborious problem, and no reliable method of estimation has existed up to now. In the present work it is shown how, from the position of the maximum concentration of a monodisperse admixture fallen onto the ground and released by an instantaneous point source, one can estimate the value of the vertical diffusion coefficient of the admixture in the atmosphere.

The equation of turbulent diffusion

$$\frac{dq}{dt} - w\frac{dq}{dz} + u\frac{dq}{dx} = k_x\frac{\partial^2 q}{\partial x^2} + k_y\frac{\partial^2 q}{\partial y^2} + k_z\frac{\partial^2 q}{\partial z^2}, \qquad (1)$$

where q is the volume concentration; xy is the plane of the Earth; z is the vertical coordinate; t is time; u is the horizontal wind velocity; w is the gravitational settling velocity of the admixture; k_x, k_y, k_z are the corresponding diffusion coefficients of the admixture, is solved under the following initial and boundary conditions:

$$\begin{array}{ll} \text{for } t = 0 & q = Q\delta(x)\delta(y)\delta(z-h), \\ \text{for } z = 0 & q = 0, \\ \text{for } \sqrt{x^2+y^2+z^2} \to \infty & q = 0. \end{array}$$

All coefficients entering equation (1) are assumed to be constant.

Let Q be the amount of substance released at the initial instant of time at the point with coordinates $x = y = 0,\ z = h$ (in the source). Under these conditions the solution of equation (1) is written in the following form:

$$q = \frac{Q}{8\pi^{3/2}t^{3/2}\sqrt{k_x k_y k_z}} \exp\left[-\frac{w^2 t}{4k_z} - \frac{w(z-h)}{2k_z} - \frac{(x-ut)^2}{4k_{zt}} - \frac{y^2}{4k_{yt}}\right] \times$$

$$\times \left\{ \exp\left[-\frac{(z-h)^2}{4k_{zt}}\right] - \exp\left[-\frac{(z+h)^2}{4k_{zt}}\right] \right\}. \tag{2}$$

The concentration on the Earth' s surface q^* is determined from the expression:

$$q^* = \int_0^\infty k_z \left.\frac{\partial q}{\partial z}\right|_{z=0} dt. \tag{3}$$

Macdonald' s function

$$K_n(x) = \frac{1}{2}\int_0^\infty e^{-x\,\mathrm{ch}\,\eta - n\eta}\, d\eta. \tag{4}$$

Using (4), we obtain

$$q^* = \frac{Qh}{4\pi\sqrt{k_x k_y k_z}} \frac{\sqrt{u^2/k_x + w^2/k_z}}{x^2/k_x + y^2/k_y + h^2/k_z} \times$$

$$\times \left(1 + \frac{2}{\sqrt{(x^2/k_x + y^2/k_y + h^2/k_z)(u^2/k_x + w^2/k_z)}}\right) \times$$

$$\times \exp\left[\frac{1}{2}\left(\frac{xu}{k_x} + \frac{hw}{k_z}\right) - \sqrt{\left(\frac{x^2}{k_x} + \frac{y^2}{k_y} + \frac{h^2}{k_z}\right)\left(\frac{u^2}{k_x} + \frac{w^2}{k_z}\right)}\right],$$

or, putting $y = 0$, we shall have

$$q^* = \frac{Qh}{4\pi\sqrt{k_x k_y k_z}} \frac{\sqrt{u^2/k_x + w^2/k_z}}{x^2/k_x + h^2/k_z} \left(1 + \frac{2}{(xu/k_x + hw/k_z)\sqrt{1+Y}}\right) \times$$

$$\times \exp\left[\frac{1}{2}\left(\frac{xu}{k_x} + \frac{hw}{k_z}\right)(1 - \sqrt{1+Y})\right], \tag{5}$$

where

$$Y = \frac{(hu - xw)^2}{k_z k_x (xu/k_x + hw/k_z)^2}.$$

From (5) we determine the position of the maximum of the surface concentration $(\partial q^*/\partial x = 0)$

$$-\frac{4k_x k_z x_{\max}}{k_z x_{\max}^2 + k_x h^2}\left(1+\frac{2}{(x_{\max}/k_x + hw/k_z)\sqrt{1+Y}}\right)-$$

$$-\frac{4x_{\max}}{(u^2/k_x + w^2/k_z)^{1/2}(x_{\max}^2/k_x + h^2/k_z)^{3/2}}+\left(1+\frac{2}{(x_{\max}u/k_x + hw/k_z)\sqrt{1+Y}}\right)\times$$

$$\times\left[u-\frac{x_{\max}\sqrt{u^2/k_x + w^2/k_z}}{\sqrt{x_{\max}^2/k_x + h^2/k_z}}\right]=0. \tag{6}$$

Let us investigate the dependence of the position of the concentration maximum on the diffusion coefficients of the impurity. It is obvious that the position of the maximum does not depend on the coefficient k_y.

It follows from expression (6) that, for $k_z \to 0$,

$$x_{\max} = x_k = \frac{uh}{w};$$

x_k is the position of the maximum in the absence of vertical diffusion.

In the limiting case, when $k_x \to \infty$,

$$X_m = \frac{x_{\max}}{x_k} = \frac{1 + 2k_z/wh}{12(k_z/wh)^2 + 6k_z/wh + 1}; \tag{*}$$

$$k_z = wh\,\frac{-3X_m + 1 + \sqrt{1 + 6X_m - 3X_m^2}}{12X_m}. \tag{7}$$

For $k_z \to \infty$, $X_m = 0$.

Let us determine the position of the maximum for small k_x. For this purpose we expand $\sqrt{1+Y}$ in a series in integer powers of Y. Comparing the third term of the expansion with the second, we obtain

$$Y \ll \frac{1}{8X^2}\frac{w^2 k_x}{u^2 k_z}(1-X)^2 \ll 1. \tag{8}$$

Inequality (8) is valid when

$$\frac{k_x w^2}{k_z u^2} \ll 0.4; \qquad X \geq 0.4; \qquad \frac{k_z}{wh} \ll 1.$$

In this case expression (5) is rewritten as follows:

Fig. 1

Figure 1: Fig. 1

$$q^* = \frac{Q}{4\pi\sqrt{k_y k_z}} \frac{w^2}{uh} \frac{\sqrt{1 + w^2 k_x/u^2 k_z}}{X^2 + w^2 k_x/u^2 k_z} \exp\left[-\frac{\frac{wh}{k_z}(1-X)^2}{4\left(X + w^2 k_x/u^2 k_z\right)}\right], \qquad \text{(A)}$$

and the position of the maximum is determined from the equation

$$X^4 + aX^3 + bX^2 + cX + d = 0, \qquad (9)$$

where

$$a = \frac{8k_z}{wh} + \frac{2w^2 k_x}{u^2 k_z}; \qquad b = \frac{w^2 k_x}{u^2 k_z}\left(16\frac{k_z}{wh} - 1\right) - 1;$$

$$c = 2\left(\frac{w^2 k_x}{u^2 k_z}\right)^2 \left[\frac{4k_z}{wh} + 1\right]; \qquad d = -\frac{k_x w^2}{k_z u^2}\left(2\frac{k_x w^2}{k_z u^2} + 1\right).$$

Fig. 1. 1–$k_x \to 0$; 2–$k_x \to \infty$ according to formula (11); 3–$k_x \to \infty$ according to formula (*);
4–$\frac{w^2 k_x}{u^2 k_z} = 0.05$; 5–$\frac{w^2 k_x}{u^2 k_z} = 0.1$; 6–$\frac{w^2 k_x}{u^2 k_z} = 0.2$; 7–$\frac{w^2 k_x}{u^2 k_z} = 0.4$

For $k_x \to 0$ we shall have

$$X^2 + \frac{8k_z}{wh} X - 1 = 0.$$

Hence

$$X_m = -\frac{4k_z}{wh} + \sqrt{\frac{16k_z^2}{w^2 h^2} + 1}. \qquad (10)$$

An analogous result can be obtained from equation (1), assuming in it $k_x = 0$, and, consequently, the flux

$$k_z \left.\frac{\partial q}{\partial z}\right|_{z=0} = \frac{Q}{4\pi t^2 \sqrt{k_y k_z}} \exp\left[-\frac{y^2}{4k_{yt}} - \frac{(h-wt)^2}{4k_{zt}}\right] \delta(x - ut).$$

Expression (*) under the assumption that $k_z/wh < 0.4$ is rewritten as follows:

$$X_m \simeq \frac{1}{4k_z/wh + 1}; \tag{11}$$

$$k_z \simeq wh \frac{1 - X_m}{4X_m}. \tag{12}$$

It follows from (10) that, for $k_x \to 0$,

$$k_z = wh \frac{1 - X_m}{4X_m} \frac{1 + X_m}{2}. \tag{13}$$

It follows from (12) and (13) that

$$(k_z)_{k_x \to 0} \simeq (k_z)_{k_x \to \infty} \frac{1 + X_m}{2}.$$

From Fig. 1, calculated according to formulas (7), (9), and (12), it follows that for $k_x w^2 / k_z u^2 \leqslant 0.4$ the value of k_z can be determined from (7) with an accuracy of up to 10%. However, calculation of k_z by formula (12) is practically more convenient. By formula (12), k_z can be calculated with an accuracy of up to 15-20% (the possible experimental error).

Proceeding from dimensional considerations, one may write that

$$k_z = \beta wh f\left(\frac{u}{w}\right).$$

Let us take $f(u/w) = u^2/w^2$; then $k_z = \beta u^2 h/w$. In this case it follows from (12) that

$$\beta \simeq \frac{1 - X_m}{4X_m} \frac{w^2}{u^2}$$

for fixed values of X, w, and u. The coefficient β is to be determined experimentally. By formula (A), the concentration at the point of maximum was calculated for values of the parameter $k_x w^2 / k_z u^2 = 0;\ 0.020;\ 0.20$. It turns out that when the parameter $k_x w^2 / k_z u^2$ changes from 0 to 0.020, the concentration changes within the range of 5-8%; when $k_x w^2 / k_z u^2$ changes from 0.02 to 0.20, it changes within the range of 15-20%.

Thus, a simple relation (12) has been established between the vertical diffusion coefficient of an impurity in the atmosphere and the position of the maximum surface concentration, and it has been shown that the magnitude of the coefficient k_x has only a slight effect both on the position of the maximum surface concentration and on its value at the point of maximum.

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Note: Figure translations are in progress. See original paper for figures.