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Abstract

Full Text

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RADIATION OF A CHARGED PARTICLE FLYING NEAR A METALLIC SCREEN

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PHYSICS

1. In the present work we consider the radiation of a point charged particle associated with the diffraction of the particle' s electromagnetic field by a metallic screen. Problems of this kind were previously considered for the case of slow ($\beta = v \ll 1,\ c = 1$) and fast ($1-\beta \ll 1$) particles in papers ([1]). The case examined below—the screen is an ideally conducting half-plane and the trajectory of the particle is perpendicular to the edge of the screen—is of interest because it makes it possible to find an exact solution of the problem for arbitrary β. Since the radiation losses are small, one may assume that the particle moves with constant velocity.

2. Let us place the z-axis along the edge of the screen, and the Ox-axis ($x > 0$) in the plane of the screen (Fig. 1). We denote by θ the angle of inclination of the trajectory to the plane, and by a the minimum distance of the particle from the edge of the screen. In what follows it is convenient to use the electromagnetic potentials $A^{(0)}$ and A, where $A^{(0)} = \{\mathbf{A}^{(0)}, \varphi^0\}$ is the field of a point particle in free space, and $A = \{\mathbf{A}, \varphi\}$ is the field induced by the charges and currents of the screen. The Lorentz gauge of the potentials is

$$\dot{\varphi} + \operatorname{div} \mathbf{A} = 0.$$

We write the solutions of Maxwell' s equations for the Fourier components $A_\omega(q) = \{\mathbf{A}_\omega(q), \varphi_\omega(q)\}$ with the aid of the charges and currents $j_\omega(q) = \{\mathbf{j}_\omega(q), \varphi_\omega(q)\}$ induced on the screen:

$$A_\omega(x,y,q) = i\pi \int_0^\infty dx'\, H_0^{(1)}\Big(p\sqrt{(x-x')^2+y^2}\Big)\, j_\omega(x'), \qquad (1)$$

$$A_\omega(x,y,q) = \frac{1}{2\pi}\int dt\, dz\, e^{i(\omega t + qz)} A(\mathbf{r},t), \qquad p = \sqrt{\omega^2 - q^2}.$$

On the screen the tangential component of the electric field vanishes. Taking into account that $\mathbf{E} = -\vec{\nabla}\varphi - \dot{\mathbf{A}}$, and using the Lorentz condition, we write the boundary condition for the scalar potential in the form (see, for example, ([2]))

$$\frac{\partial^2\overline{\varphi}}{\partial x^2} + p^2\overline{\varphi} = -\frac{\partial^2\overline{\varphi}^{(0)}}{\partial x^2} + q^2\overline{\varphi}^{(0)} + i\omega\,\frac{\partial \overline{A}_x^{(0)}}{\partial x}. \tag{2}$$

The bar above means that the values of the potentials are taken on the screen $(x > 0,\ y = 0)$. The index ω is omitted here and below. In deriving (2) one should take into account that $A_z^{(0)} \equiv 0,\ A_y \equiv 0$.

The values of the "zero" potentials on the surface of the screen are equal to

$$A^{(0)}(x) = \frac{e}{\alpha}\exp\left(-\tau x - \frac{\alpha a}{\beta}\right)\{\beta\cos\theta,\ \beta\sin\theta,\ 0,\ 1\}, \tag{3}$$

where

$$\alpha = \left|\sqrt{\omega^2\gamma^2 + q^2\beta^2}\right|, \qquad \tau = \frac{\alpha\sin\theta - i\omega\cos\theta}{\beta}, \qquad \gamma = \sqrt{1-\beta^2}.$$

Condition (2) leads to the following expressions for the additional potentials on the screen:

$$\overline{\varphi}(x) = c_1 e^{ipx} + c_2 e^{-ipx} - \frac{e}{a}\,\frac{\alpha\cos\theta + i\omega\gamma^2\sin\theta}{\alpha\cos\theta + i\omega\sin\theta}\exp\left(-\tau x - \frac{\alpha a}{\beta}\right); \tag{4}$$

$$\overline{A}_x(x) = c_1\frac{p}{\omega}e^{ipx} - c_2\frac{p}{\omega}e^{-ipx} - \frac{e\beta}{\alpha\cos\theta + i\omega\sin\theta}\exp\left(-\tau x - \frac{\alpha a}{\beta}\right), \tag{5}$$

where c_1 and c_2 are constants of integration.

3. Setting $y = 0$ in (1), we obtain, in order to determine $j(x)$, integral equations of the Wiener–Hopf type. For example, for j_x in the Fourier representation equation (1) takes the form

$$\overline{A}_x^{+}(k) + \overline{A}_x^{-}(k) = h(k) j_x(k), \tag{6}$$

$$\overline{A}_x^{\pm} = \int_0^\infty A_x(\pm x)e^{\pm ikx}\,dx, \qquad j_x(k) = \int_0^\infty e^{ikx} j_x(x)\,dx.$$

Here

$$h(k) = \frac{2\pi i}{\sqrt{p^2 - k^2}}.$$

Fig. 1

Figure 1: Fig. 1

Fig. 1

The functions with the plus and minus signs are analytic, respectively, in the upper and lower half-planes of k. The function $j_x(k)$ is analytic in the upper half-plane of k. The factorization in equation (6) is carried out as follows. Assuming that $\operatorname{Im}\omega = -\delta < 0$, we write $h(k)$ in the form $h(k) = h^+(k)/h^-(k)$, where $h^+(k) = 2\pi i/\sqrt{p-k}$ and $h^-(k) = \sqrt{p+k}$. $h^+(k)$ is analytic for $\operatorname{Im} k > -\delta$, while $h^-(k)$ is analytic for $\operatorname{Im} k > \delta$.

It remains now to split $\overline{A}_x^+(k)h^-(k)$ into two terms analytic in the upper and lower half-planes of k:

$$\overline{A}_x^+(k)h^-(k) = \lambda^-(k) - \lambda^+(k),$$

$$\lambda^\pm(k) = \frac{1}{2\pi i}\int_{-\infty\mp i\sigma}^{\infty\mp i\sigma} \overline{A}_x^-(k')h^-(k')\,\frac{dk'}{k'-k}, \qquad 0<\sigma<\delta. \tag{7}$$

Equation (6) now takes the form:

$$\lambda^-(k) + \overline{A}_x^-(k)h^-(k) = \lambda^+(k) + j_x(k)h^+(k). \tag{8}$$

Since the functions on the right- and left-hand sides of (8) have a common strip of analyticity $|\operatorname{Im} k| < \delta$, it follows that $\lambda^+(k) + j_x(k)h^+(k) \equiv P(k)$, where $P(k)$ is an entire function. As $k \to \infty$,

$$\overline{A}_x^+(k) \sim 1/k, \qquad h^+(k) \sim 1/\sqrt{k}, \qquad \lambda^+(k) \sim 1/k, \qquad j(k) \to 0.$$

Since $P(k)$ is regular for finite k, it is necessary to set $P(k) \equiv 0$. Hence we find that $j_x(k) = -\lambda^+(k)/h^+(k)$. From (5) and (7) we have

$$\lambda^+(k) = \frac{1}{\sqrt{2\pi}}\left[\frac{ic_2\sqrt{2p}}{p-k} - \frac{ie\beta\sqrt{p-i\tau}}{\alpha\cos\theta + i\omega\sin\theta}\,\frac{1}{k+i\tau}\right]. \tag{9}$$

From the condition $j_x(x=0) = 0$ we determine c_2. Finally, for $j_x(k)$ we have the expression

$$j_x(k) = B_{q\omega}\,\frac{p+i\tau}{\sqrt{p-k}\,(k+i\tau)}, \tag{10}$$

$$B_{q\omega} = \frac{1}{4\pi^2} \frac{e\beta\sqrt{p - i\tau}}{\alpha\cos\theta + i\omega\sin\theta} \exp\left(-\frac{\alpha a}{\beta}\right).$$

In an analogous way, the Fourier component of the charge density can be found,

$$\rho(k) = -B_{q\omega}\left(\frac{\omega}{p}\frac{1}{\sqrt{p-u}} - \frac{\alpha\cos\theta + i\omega\gamma^2\sin\theta}{\alpha\beta}\frac{\sqrt{p-k}}{k+i\tau}\right). \tag{11}$$

Thus, the potentials $A_x(x, y)$ and $\varphi(x, y)$ have been determined. To find $A_z(x, y)$ it is sufficient to use the Lorentz condition.

4. We now proceed to the calculation of the total radiated energy W

$$W = \frac{R^2}{4\pi}\int_{-\infty}^{\infty} d\omega \int d\Omega\,[\mathbf{E}_\omega(R)\mathbf{H}_\omega(R)]_R. \tag{12}$$

The integration of the R-component of the Poynting vector in (12) is carried out over a sphere of large radius.

In calculating the fields in the wave zone one should take into account that the characteristic size of the currents and charges is, generally speaking, comparable with the radiation wavelength. Therefore the expressions for the asymptotics of the potentials take the form

$$A(R) = \frac{e^{i\omega R}}{R}\int d\mathbf{r}'\, e^{-i\omega R\mathbf{r}'/R}\mathbf{j}(\mathbf{r}') = \frac{2\pi e^{i\omega R}}{R}\, j(k_0, q_0), \tag{13}$$

where $k_0 = -\omega\sin\psi\cos\varphi$, $q_0 = -\omega\cos\psi$. Here a spherical coordinate system has been used: $x = R\sin\psi\cos\varphi$, $y = R\sin\psi\sin\varphi$, $z = R\cos\psi$.

Now expression (12) will take the form

$$W = 2\pi\int_0^\infty d\omega\,\omega^2\int_0^\pi\int_0^{2\pi} d\psi\,d\varphi\,\sin\psi\left\{|j_x(k_0, q_0)|^2\sin^2\varphi + \right.$$
$$\left. + |j_x(k_0, q_0)\cos\varphi - \rho(k_0, q_0)\sin\psi|^2\frac{1}{\cos^2\psi}\right\}. \tag{14}$$

After integration over the frequencies we obtain

$$W = \frac{e^2\beta^2}{4\pi^2 a}\int_0^\pi\int_0^{2\pi} d\psi\,d\varphi\,\frac{\cos^2\varphi/2\,\cos^2\psi\,(1-\beta\sin\psi\cos\theta) + \alpha_0^2\sin^2(\varphi/2)(1+\beta\sin\psi\cos\theta)}{\alpha_0^3|p_0\cos\varphi - i\tau_0|^2},$$

$$\alpha_0 = \alpha(q_0)/|\omega|, \qquad p_0 = p(q_0)/\omega, \qquad \tau_0 = \tau(q_0)/\omega. \tag{15}$$

Finally, after integration over the angles we ultimately find

$$W = \frac{3}{8}\frac{e^2\beta^2}{a\gamma}. \tag{16}$$

5. Thus, the total radiated energy in both limiting cases, for small and large velocities, proves to be proportional to the kinetic energy of the particle. This result differs from the estimates given in (1), where it was shown that at small particle velocities $W \sim \beta^3$. The difference is connected with the fact that in the present case it is incorrect to take the size of the effectively radiating dipole to be of order a. As was already noted, in this case the current decreases over distances of the order of the wavelength, i.e. a/β, so that the estimate $W \sim \beta^3$ is valid only for screens with dimensions of order a. For $\beta \ll 1$ the pattern of the radiation field does not depend on the inclination of the particle trajectory, but is determined entirely by the boundary conditions. In this case the spectral density of radiation is $W(\psi, \varphi) \sim \cos^2(\varphi/2)\cos^2\psi + \sin^2(\varphi/2)$.

In the ultrarelativistic case, when the characteristic wavelength is small in comparison with a, the problem can be considered in the approximation of geometrical optics. In this case the losses to radiation are equal to twice (since the radiation "forward" and "backward" is the same) the energy of the electromagnetic field of a free particle contained in the region of the geometrical shadow from the screen. The boundary of the shadow, obviously, coincides with the surface drawn

through the edge of the screen parallel to the particle trajectory. Indeed, the quantity

$$\int\limits_V \frac{(\mathbf{E}^0)^2 + (\mathbf{H}^0)^2}{4\pi}\, dV,$$

where V is the region of the geometrical shadow, coincides with (16) for $1-\beta \ll 1$. In this way, in the ultrarelativistic case one can find the radiation losses for a screen of arbitrary shape.

Let us note in conclusion that the radiation considered is practically appreciable for bunches of electrons (the losses grow in proportion to the square of the number of particles in the bunch). The results of the work remain applicable if the dimensions of the bunch are small in comparison with a.

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Note: Figure translations are in progress. See original paper for figures.