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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

MECHANICS OF CONTINUOUS MEDIA

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SOME CONCEPTS CONCERNING THE ZONE AND FRONT OF CRACKS

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One of the properties of solid bodies is the ability to fracture, forming cracks and ruptures within themselves. A distinctive feature of many nonmetallic solids is their low tensile strength in comparison with their compressive strength. A description of motion in a solid medium in which discontinuities of continuity (cracks) arise can hardly be given in general form at the present time. It is therefore useful to consider certain particular cases of such motion and, first of all, those of practical significance.

In this article we shall consider spherically symmetric motion in a medium that is less strong in tension than in compression. The basic ideas used here will evidently also be useful in considering other cases.

At some instant, on some sphere inside a solid body, forces begin to act that generate a spherical elastic wave. As this wave propagates in the solid body, both compressive and tensile stresses arise. The greatest tensile stress in this case occurs primarily in the azimuthal direction. This stress may exceed the critical value, and then rupture will occur; a crack will form in the radial direction. Such cracks densely penetrate a certain volume, which we shall call the crack zone, located at some distance behind the front of the elastic wave. The line that separates this zone from the elastic zone will be called the crack front. Before the cracks form, the motion in the solid body is an elastic wave, is described by a single function—the elastic potential—and is completely determined by the boundary condition on the inner cavity.

Fig. 1

In Fig. 1, which represents the picture of the motion in the plane r, t (r is the distance from the center, t is time) and a section in physical space at some instant of time, this is region 1. (It is bounded by the front of the elastic wave, i.e., by the characteristic from the initial point of action on the body, and by the characteristic departing from the cavity at the instant of formation of the crack zone t_*.) From the instant of formation of the crack zone, the motion ahead of the crack front, in zone 2, will be an elastic wave described, likewise,

as in region 1, by a single function—the elastic potential, which must be determined from the conditions at the crack front. Thus, in order to solve the problem completely, we must consider the crack zone and formulate the conditions at the crack front.

The basic assumptions are as follows:

I. The motion in the crack zone remains spherically symmetric.

II. The cracks that arise, being distributed uniformly in the crack zone, completely and instantaneously relieve the azimuthal stress.

III. Cracks form at the moment when the azimuthal stress in the elastic wave reaches the critical (rupturing) stress characteristic of the given solid. (For many practically interesting cases the critical stress is close to zero.)

The first two hypotheses can, to a certain extent, be justified by the symmetry and constrained nature of the motion, which are imposed by the geometry, by the small deformations in the elastic solid, and by the spherical elastic wave. The elastic wave, as it were, implants or produces cracks. Let us note at once that the cracks influence the elastic wave not directly, but by regulating the transmission of the action from the initial sphere to the elastic wave through the crack zone. If the direct influence of a crack on the elastic wave is small, then the crack is formed anew all the time, as it were. Hence the third hypothesis also becomes understandable.

A mathematical description of the crack zone is now easily obtained if we consider radial elastic motion from the center within some solid angle and assume that the free boundaries (the boundaries of this solid angle) instantaneously relieve the azimuthal stress. Where the small cracks densely penetrate the volume, they effectively unload the azimuthal stress. They are formed, if one may so express it, precisely in order to unload it. Such is the model chosen for the present solution. It will be a good one for media in which, in reality, a network of radial cracks forms*. For some media this model may be modified by introducing friction (for example, for crushed and initially fractured rocks), but if the symmetry and unloading remain, the problem is still easily solved.

From the very beginning of the formation of the crack zone, at the crack front, i.e., on the surface connecting the initial points of the cracks, all quantities (stress, velocity, density) in the radial direction will be discontinuous.

After these basic considerations, let us write down the equations. In zones 1 and 2 the motion is a running elastic wave and is determined by the elastic potential $\psi\left(t - \frac{r - r_0}{c}\right)$, where c is the velocity of longitudinal elastic waves.

The displacement, velocity, radial and azimuthal stresses are expressed in the following way in terms of ψ (a dot denotes differentiation with respect to the argument, σ is Poisson' s ratio, ρ is the density):

$$u_r = \frac{\dot{\psi}}{cr} + \frac{\psi}{r^2}; \quad (1)$$

$$v_r = \frac{\ddot{\psi}}{cr} + \frac{\dot{\psi}}{r^2}; \quad (2)$$

$$-\frac{\sigma_r}{\rho c^2} = \frac{\dddot{\psi}}{c^2 r} + \frac{2(1-2\sigma)}{(1-\sigma)}\left(\frac{\dot{\psi}}{cr^2} + \frac{\psi}{r^3}\right); \quad (3)$$

$$-\frac{\sigma_\theta}{\rho c^2} = \frac{\sigma}{(1-\sigma)}\frac{\dddot{\psi}}{c^2 r} - \frac{(1-2\sigma)}{(1-\sigma)}\left(\frac{\dot{\psi}}{cr^2} + \frac{\psi}{r^3}\right). \quad (4)$$

* If, behind the elastic wave, there remains a very large static pressure, then individual cracks will develop quasistatically from the crack zone. This phenomenon requires special consideration; it is not covered by the present model. However, it practically does not affect the elastic wave and the size of the zone densely penetrated by cracks.

Using the boundary condition (the prescribed pressure on the wall of the internal cavity), we find the form of the function $\psi(\tau)$.

The motion in zone 3 represents a traveling wave and is described by the function $f(\tau')$

$$\tau' = t - \frac{r - r_0}{c_0}; \quad (5)$$

$$c_0 = \sqrt{E/\rho}; \quad (6)$$

E is Young' s modulus.

The displacement, velocity, and radial stress are expressed as follows:

$$u_r = \frac{f(\tau')}{r}; \quad (7)$$

$$v_r = \frac{\dot{f}(\tau')}{r}; \quad (8)$$

$$-\frac{\sigma_r}{E} = \frac{\dot{f}(\tau')}{c_0 r} + \frac{f(\tau')}{r^2}. \quad (9)$$

If the pressure at the initial radius is specified, we find the form of the function $f(\tau')$.

Thus, we know the solution for zone 3, but we do not know how far this zone extends, i.e., we do not know the law of motion of the crack front and we do not know the function determining the motion in region 2. The law of motion of the crack front $R(t)$ and the elastic potential for region 2, $\psi_2(\tau)$, are determined from the conditions at the crack front

$$\sigma_{r_2} = \sigma_{r_3}; \tag{10}$$

$$\sigma_{\theta_2} = \sigma_{\text{cr}}. \tag{11}$$

These conditions lead to a system of two ordinary differential equations for R and ψ_2.

Thus, for any particular case the problem can be solved completely. (It should be noted that the theory is applicable only in the case $d^2R/dt^2 < 0$. In other words, this theory describes motion with a decelerating crack front.)

Some general effects can be estimated. It is seen that the established (i.e., as $t \to \infty$) stress distribution along the radius is different for the case of a continuous medium and a medium separated by cracks. In the case of a continuous medium, the stress decreases as the cube of the distance; in the case of a medium separated by cracks, as the square. It follows that radial cracks facilitate the transmission of stress over large distances. This action in the wave leads, in particular, to an increase in the wave period and to an increase in the emitted elastic energy. In reality the crack front lags considerably behind the elastic-wave front and, by the end of its motion, is practically in the region of statics. Therefore, for the final radius of the crack zone R_{k}, we obtain the following expression:

$$R_{\text{k}} = r_0\sqrt{\frac{P_{\text{k}}}{2\sigma_{\text{cr}}}}, \tag{12}$$

P_{k} is the pressure in the cavity at the moment when quasistatics is established.

For particular problems, an approximate solution can be found if the velocity and acceleration of the crack front at the moment of its formation are calculated.

At the All-Union meeting of the Council on the National-Economic Use of Explosions in Novosibirsk in March 1961, some experimental results and calculation results were reported. These results are in good agreement.

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Note: Figure translations are in progress. See original paper for figures.