# ON THE FORMULATION OF THE PROBLEM OF THE STABILITY OF MINE WORKINGS

1962

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## Abstract

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### CONTINUUM MECHANICS

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# ON THE FORMULATION OF THE PROBLEM OF THE STABILITY OF MINE WORKINGS

*(Presented by Academician A. Yu. Ishlinskii, February 27, 1961)*

In the work (1) the question was investigated of determining the state of stress in a rock mass near a vertical cylindrical working of circular cross section, one end of which opens onto the ground surface. It was assumed that the mass is homogeneous, isotropic, and obeys the laws of the theory of elasticity. Under these assumptions, in the cylindrical coordinate system $r, \theta, z$, the following expressions were obtained for the components of stresses and displacements:

$$\sigma_r = -\frac{k\mu}{1-\mu}\, z\left(1-\frac{a^2}{r^2}\right), \qquad \sigma_\theta = -\frac{k\mu}{1-\mu}\, z\left(1+\frac{a^2}{r^2}\right), \qquad \sigma_z = -kz, \quad \tau_{rz} = 0; \tag{1}$$

$$u = -\frac{k\mu(1+\mu)}{E(1-\mu)}\, z\,\frac{a^2}{r}, \qquad w = \frac{k(1+\mu)}{E(1-\mu)}\left[\mu a^2 \ln r - \frac{1-2\mu}{2}\, z^2 + c\right], \tag{2}$$

where $a$ is the radius of the cross section of the working; $k$ is the weight of a unit volume of the mass; $E, \mu$ are the modulus of elasticity and Poisson' s ratio of the mass. The constant $c$ depends only on the initial plane from which displacements are measured. Formulas (1), (2) may be used, for example, to characterize the stressed and deformed state in rocky strata in those cases where supporting timbering is not required.

Below the question of the loss of stability of a working of this kind as an elastic body is studied; in other words, relations are sought between the parameters of the mass and of the working under which buckling of the latter is possible. The proposed solution is approximate in character, and it should be regarded as one of the possible attempts to study theoretical problems in the mechanics of rocks. Under real conditions, during collapses in mines, the rock mass passes into a plastic state. Taking this circumstance into account introduces additional difficulties and is not considered here.

One approach to the consideration of the problem of the stability of mine exposures is the use of methods for solving stability problems of elastic equilibrium from the standpoint of the general equations of the theory of elasticity. These

Fig. 1

Figure 1: Fig. 1

methods were developed in the works ($^2$, $^3$), which are based on the following assumptions. The stresses and displacements of a certain elastic body are assumed to consist of initial components, which are hereinafter called the components of the basic unperturbed state, and of certain additional ones, the appearance of which may lead to a violation of the uniqueness of the solutions of the problem of the theory of elasticity. These additional terms (components of perturbation) are considered so small that their powers higher than the first and their products may be neglected in comparison with the products of these quantities by the initial components. Those values of the external loads (called critical) are sought at which several forms of elastic equilibrium of the body may exist; boundary conditions are used for this purpose. The equations of equilibrium, the relations between stresses and strains, and those between strains and displacements for the perturbation components are taken to be the same as for

initial ones. Thus, it is assumed that the loss of stability of elastic equilibrium occurs mainly through a change in the boundary conditions. In a more general formulation these problems were investigated in work ($^4$).

Let us consider a massif with a vertical cylindrical excavation of circular cross section and depth $H$ (Fig. 1). Neglecting the influence of edge conditions, for an excavation of finite depth we shall characterize the stressed state in the massif by expressions (1). At some depth $h$ ($h < H$), let us single out an annular layer of the massif of height $l$. It is obvious that this layer is under the action of a uniform pressure $p = kh$. We shall neglect the influence of the weight of the annular layer in comparison with the acting forces.

Fig. 1

We shall seek the solution of the problem in the form:

$$\sigma_r = \sigma_r^0 + \sigma_r', \ldots, \quad u = u^0 + u', \ldots \tag{3}$$

The stressed and deformed state of the annular layer before loss of stability will be specified by the formulas:

$$\sigma_r^0 = A\left(1 - \frac{a^2}{r^2}\right), \quad \sigma_\theta^0 = A\left(1 + \frac{a^2}{r^2}\right),$$

$$\sigma_z^0 = -p, \quad \tau_{rz}^0 = 0; \tag{4}$$

$$u^0 = -B\mu h\,\frac{a^2}{r},$$

$$w^0 = B\left[\mu a^2 \ln r - \frac{1-2\mu}{2} h^2 + c\right], \tag{5}$$

where

$$A = -kh\mu/(1-\mu), \quad B = k(1+\mu)/E(1-\mu), \quad p = kh.$$

Thus, the solution of the problem reduces to determining such a value $p = p_{\text{cr}}$ at which the components of the perturbed state $(\sigma_r', \sigma_\theta', \ldots)$ may be different from zero. For this purpose it is necessary to make use of the equilibrium equations, the relations of the law of elastic deformation, and the boundary conditions. According to work ($^5$), one may obtain

$$u' = \left[(3-4\mu)f(r) - \frac{r}{a}f'(r) - g'(r)\right]\cos\beta z,$$

$$w' = [rf(r) + ag(r)]\beta\sin\beta z, \tag{6}$$

where

$$f(r) = C_1 I_1(\beta r) + C_2 K_1(\beta r), \quad g(r) = D_1 I_0(\beta r) + D_2 K_0(\beta r), \tag{7}$$

where $\beta$ is the buckling parameter; $I_\nu(\beta r)$ and $K_\nu(\beta r)$ are, respectively, the Bessel and Macdonald functions of imaginary argument. The components of the stresses of the perturbed state are determined from the relations of the law of elastic deformation, taking (6) into account.

For what follows it is necessary to have:

$$\sigma_r' = 2G\left\{(3-2\mu)f'(r) - \left[\beta^2 r + (1-2\mu)\frac{1}{r}\right]f(r) + \frac{a}{r}g'(r) - a\beta^2 g(r)\right\}\cos\beta z,$$

$$\tau_{rz}' = 2G\left[rf'(r) - (1-2\mu)f(r) + ag'(r)\right]\beta\sin\beta z. \tag{8}$$

($G$ is the shear modulus.)

Since the components of the perturbed state must tend to zero at infinity, in formulas (7) one should set $C_1 = D_1 = 0$.

The boundary conditions are written in the form ($^6$)

$$\sigma_r' + \frac{d\sigma_r^0}{dr}u' = 0, \quad \tau_{rz}' + \sigma_z^0\frac{du'}{dz} = 0 \quad \text{at } r = a. \tag{9}$$

Fig. 2

Figure 2: Fig. 2

Substituting (6) and (8) into (9), taking (4) and (7) into account, we obtain a homogeneous linear system of algebraic equations with respect to $C_2$ and $D_2$. At loss of

for the stability of the working this system must have a nontrivial solution; therefore its determinant must be equal to zero. Hence we obtain the relation between the critical parameters in the form

$$\gamma=\frac{2(1-\mu)K_1^2(\alpha)+\alpha^2\left[K_1^2(\alpha)-K_0^2(\alpha)\right]}{4\mu K_1^2(\alpha)+\alpha^2\left[K_1^2(\alpha)-K_0^2(\alpha)\right]-2(1-\mu)\alpha K_0(\alpha)K_1(\alpha)}. \tag{10}$$

Here the notation $\gamma=kh/2G,\ \alpha=\beta a$ has been introduced.

The initial height of the annular layer $l$ is determined from the condition that at its ends the displacement is equal to zero. Using (6), we obtain $l=\pi/2\beta$. For the Macdonald functions $K_\nu(\alpha)$, the following formulas hold as $\alpha\to 0$:

$$K_\nu(\alpha)\underset{\alpha\to 0,\ \nu>0}{\approx}\frac{2^{\nu-1}\Gamma(\nu)}{x^\nu},$$

$$K_0(\alpha)\underset{\alpha\to 0}{\approx}\ln\frac{2}{\alpha}, \tag{11}$$

where $\Gamma(\nu)$ is the gamma function.

The asymptotic behavior of the function $K_\nu(\alpha)$ as $\alpha\to\infty$ is represented by the equalities

$$K_\nu(\alpha)\approx\left(\frac{\pi}{2\alpha}\right)^{1/2}e^{-\alpha}. \tag{12}$$

**Fig. 2**

Using (11) and (12), as well as the known techniques of differential calculus, it is not difficult to investigate the function $\gamma=\gamma(\alpha)$ specified by expression (10), and to plot its graph. For $\mu=0.2$, Fig. 2 gives the graph $\gamma=\gamma(\alpha)$ (curve $\gamma_1$).

It is also necessary to plot the graph of $\gamma$ versus $\alpha$, proceeding from the relation

$$\gamma=\frac{k}{2G}\left(H-\frac{\pi a}{2\alpha}\right). \tag{13}$$

The point of intersection of the two graphs gives the critical value of the parameter $\gamma_{\text{cr}}$. Thus, the rocks of the working walls will be stable if the condition

$\gamma \leqslant \gamma_{\text{cr}}$ is satisfied. Let us note that, as is seen from (13), the radius of the working has a substantial influence on its stability.

Let us consider a numerical example for the following data: $k = 2.6$ t/m$^3$, $G = 2.6 \cdot 10^5$ kg/cm$^2$, $\mu = 0.2$, $H = 1000$ m, $a = 4$ m. According to (13), we construct the graph $\gamma = \gamma(\alpha)$ (curve $\gamma_2$ in Fig. 2), whence we obtain $\gamma_{\text{cr}} \approx 0.5 \cdot 10^{-3}$. For the selected dimensions of the vertical shaft and the mechanical characteristics of the rock, the stability condition is satisfied.

The author expresses gratitude to A. Yu. Ishlinsky for his attention to the work and for his substantial comments on it.

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Received
26 II 1961

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