EXPERIMENTAL STUDY OF THE STABILITY OF CYLINDRICAL SHELLS BEYOND THE ELASTIC RANGE

`![Fig. 1](image)`

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

THEORY OF ELASTICITY

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EXPERIMENTAL STUDY OF THE STABILITY OF CYLINDRICAL SHELLS BEYOND THE ELASTIC RANGE

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Problems on the stability of isotropic shells within the elastic range are at present considered chiefly as problems of buckling "in the large," i.e., from the point of view of nonlinear theory ([1], [2]). As is known, such an approach makes it possible to explain the considerable scatter of experimental values of critical loads and the sharp decrease of actual critical stresses in comparison with the "classical" formulas of the linear theory. The question arises: to what extent does such a statement apply to the inelastic range? In other words, in studies of the stability of shells beyond the elastic limit, should one take into account not only the "physical" but also the "geometrical" nonlinearity of the problem? From the very beginning it is clear that, for a very small exceedance of the elastic limit, geometrical nonlinearity must retain its role; however, to what extent is this true when plastic deformations have developed? In order to answer these questions, the authors of the article carried out an experimental study of the stability, within and beyond the elastic range, of closed circular cylindrical shells subjected to axial compression.

a *b* *c*

Fig. 1

Thirty specimens were tested, made of duralumin alloy D16T; the ratio of the length L to the radius of the middle surface R was

$$L/R = 2.$$

The ratio of the radius to the shell thickness R/h varied from 25 to 135. The shells were turned from tubes on a lathe, with increased requirements imposed on the manufacturing technology. Loading of the shells was carried out on a

Fig. 2

Figure 2: Fig. 2

hydraulic press; measures were taken to ensure that the load was distributed uniformly over the cross section.

All specimens can be divided into three series. The first series included shells with the ratio R/h from 80 to 130, which underwent buckling in the elastic range. Loss of stability here occurred in the form of a sharp snap, during which rhombus-shaped dents were formed; in outline they were slightly elongated along the arc (Fig. 1*a*). The second series included shells for which the ratio R/h was from 35 to 80. The compressive stress present at buckling exceeded the elastic limit of the material. Loss of stability for these shells was also accompanied by the rapid appearance of dents, but the snap was smoothed out. The dents likewise had a rhombic character, but were already strongly elongated along the arc (Fig. 1*b*). The number of waves formed along the circumference, conseq-

···decreased with decreasing R/h. Finally, the third series was represented by shells with a comparatively thick wall; the ratio R/h was 25–35. For such shells the phenomenon of loss of stability consisted in the formation of a continuous annular bulge (Fig. 1b).

Figure 2 shows the dependence of the critical stress on the ratio R/h. Also plotted here are curves obtained as a result of an approximate theoretical solution of the geometrically linear problem. If one uses the relations following from deformation theory ([3]), and conditionally assumes that the material is incompressible and that there is no unloading zone, then the following formula ([4]) can be obtained for the upper critical stress:

$$p_+ = \frac{2}{3}\sqrt{E_t E_s}\,\frac{h}{R}, \tag{1}$$

where E_t is the tangent modulus, and E_s is the secant modulus. In work ([5]) this dependence is generalized to the case of a compressible material.

Fig. 2

Proceeding from the theory of flow, we arrive at a formula analogous to (1), with E_s replaced by the basic modulus E. For the elastic region one should set $E_s = E_t = E$.

In Fig. 2, curves 1 and 2 correspond to the solution according to deformation theory (1 –allowing for compressibility at $\mu = 0.32$, 2 –for $\mu = 0.5$); curve 3 corresponds to the solution according to the theory of flow. The dashed parts of the lines refer to the elastic region.

In Fig. 3 the same results are presented in another form: along the ordinate axis is plotted the dimensionless stress parameter $\bar{p}_+ = p_+ R/Eh$. For $R/h > 80$ the

Fig. 3

Figure 3: Fig. 3

mean experimental value $\bar{p}_+$, lying within the elastic range, was 0.27, whereas the upper theoretical value for the elastic region is 0.61 for $\mu = 0.32$ and 0.67 for $\mu = 0.5$; the reduction reaches 55-60%. Meanwhile, as is seen from Figs. 2 and 3, in the elastic-plastic region this discrepancy is gradually smoothed out. Thus, for example, at the critical strain $\varepsilon = 4 \cdot 10^{-3}$, which is 4/3 of the limiting elastic strain ε_e, the reduction in comparison with (1) is about 20%. At considerable plastic deformation the experimen—

Fig. 3

The experimental values of the critical stresses are very close to the data obtained from the geometrically linear theory.

Thus, the effect of geometric nonlinearity, which plays a primary role in problems of the elastic stability of shells, is gradually smoothed out in the plastic range. At developed plastic strains, exceeding in the case of duralumin $1.7\varepsilon_e$, the nonsymmetric buckling mode passes into an axisymmetric one. For relatively thick shells, the stability calculation can apparently be carried out by considering only buckling "in the small." Of greatest interest is the intermediate region, where it is necessary to take into account simultaneously the "physical" and "geometric" nonlinearity of the problem.

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Note: Figure translations are in progress. See original paper for figures.