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Abstract

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ON THE ACCELERATION OF PARTICLES BY RADIATION IN THE PRESENCE OF A MEDIUM

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1. The acceleration of particles by a directed flux of radiation in vacuum is relatively small because of the small value of the Thomson scattering cross section ([1]). Thus, even at a radiation pressure of the order of 10 atm, the acceleration amounts to only 10^{-7} eV per 1 cm of path. It should be noted, however, that the presence of a medium can fundamentally change the situation if the radiation density exceeds a certain critical value. Indeed, in the presence of a medium there arises the possibility of Cherenkov radiation and absorption of radiation quanta, the probabilities of which are proportional to the square of the charges, and not to e^4, as in the case of scattering. The probability of absorption of a quantum is proportional to $N_{\omega,\mathbf{k}}$, while the probability of emission is proportional to $(N_{\omega,\mathbf{k}} + 1)$, where $N_{\omega,\mathbf{k}}$ is the number of quanta of frequency ω and momentum $\mathbf{k}$. The acceleration of a particle is found as the difference between the absorbed and emitted energies. If, in the probabilities of emission and absorption, the quantum recoil effect is neglected ([2]), as was done in ([3]), then in the indicated difference $N_{\omega,\mathbf{k}}$ cancels out and only the effect of spontaneous Cherenkov radiation remains, leading to deceleration of the particle. However, the conclusion drawn in ([2]) that the energy losses of the particle do not depend on the radiation density is incorrect, since allowance for the recoil effect leads to the fact that $N_{\omega,\mathbf{k}}$ does not cancel. The result in the nonquantum limit is not equal to zero, since it is proportional to the classical quantity of the radiation density ρ.

The possibility of correctly allowing for the quantum recoil effect ([2]) in calculating the probability of Cherenkov radiation with the aid of so-called phenomenological quantum electrodynamics, which uses quantization of averaged quantities, has recently been questioned. Recent successes in the theory of many-particle systems and in the Green' s-function method make it possible to approach the solution of the problem of the quantum recoil effect for Cherenkov radiation by using only quantization of the microfields ([4]). In this way the deceleration or acceleration of a particle in a medium is found from the imaginary part of the effective energy spectrum, obtained from the poles of the analytic continuation of the particle Green' s function. The probability obtained for emission and absorption for media with spatial dispersion is valid in regions where the medium has strong absorption.

2. If spatial dispersion may be neglected, then in the region of transparency of the medium, for the acceleration (increase in energy per 1 cm of path) of a particle of spin 1/2, we obtain:

$$\frac{dW}{dx} = e^2 v \int_{\cos\theta_- < 1} \omega\, d\omega\, N(\omega, \theta_-) \left[1 - \frac{1}{n^2 v^2} + \frac{\omega}{vp}\left(1 - \frac{1}{n^2}\right) + \frac{\omega^2}{4p^2} n^2 \left(1 - \frac{1}{n^4}\right)\right] -$$

$$-e^2 v \int_{\cos\theta_+ < 1} \omega\, d\omega\, (N(\omega, \theta_+) + 1) \left[1 - \frac{1}{n^2 v^2} - \frac{\omega}{vp}\left(1 - \frac{1}{n^2}\right) + \frac{\omega^2}{4p^2} n^2 \left(1 - \frac{1}{n^4}\right)\right], \tag{1}$$

where $n^2(\omega) = \varepsilon(\omega)$; v is the velocity; p is the momentum of the particle ($\hbar = c = 1$);

$$\cos\theta_\pm = \frac{1}{nv} \pm \frac{\omega\,(n^2 - 1)}{2pn}; \qquad N(\omega, \theta) = \frac{1}{2\pi} \int_0^{2\pi} N(\omega, \theta, \varphi)\, d\varphi \quad \text{—averaged over}$$

at an angle φ, the number of quanta of frequency ω, momentum $k = \omega n/c$, and direction determined by the spherical angles θ, φ (the z axis is along p). If in (1) $N = 0$, we obtain Cherenkov braking, and the result, under the assumptions made, coincides with the result of phenomenological quantum electrodynamics*.

3. Let us consider nearly monochromatic radiation for which both the condition for Cherenkov emission and the condition for Cherenkov absorption are satisfied, and whose intensity depends only weakly on the angles**:

$$\rho(\omega) \simeq \frac{\rho}{\pi} \Delta\omega \left[(\omega - \omega_0)^2 + \Delta\omega^2\right]^{-1}, \tag{2}$$

where ρ is the total radiation density; $\rho(\omega)$ is the spectral density. Taking into account $\Delta\omega \ll \omega_0$ and the relation between $\rho(\omega)$ and $N(\omega)$, we easily find from (1) (omitting the usual Cherenkov braking):

$$\frac{dW}{dx} \simeq 2\pi r_0 \lambda_0 \rho \frac{mc^2}{E} \left[\frac{1}{n^2(\omega_0)\beta^2} \left(1 - \frac{1}{n^2(\omega_0)}\right)\right], \tag{3}$$

where $r_0 = e^2/mc^2$ is the classical radius, $\beta = v/c$; $\lambda_0 = 2\pi c/\omega n(\omega_0)$; E is the energy of the particle. One may compare (3) with the acceleration acquired by a particle in Compton scattering,

$$\frac{dW_k}{dx} = \frac{2\pi}{3} r_0^2 \rho \left(\frac{mc^2}{E} \right)^2; \qquad E \gg mc^2.$$

Taking the factor in square brackets in (3) to be a number of order unity, we obtain that (3) gives an acceleration $\frac{\lambda_0}{r_0}\frac{E}{mc^2}$ times larger (for visible light, by $\sim 10^9$ times). In order for acceleration to occur, it is necessary that (3) exceed the usual spontaneous Cherenkov braking, which is of order e^2/λ_s^2, where λ_s is a characteristic intrinsic frequency of the medium. This condition is fulfilled for

$$\rho > \rho_{\text{cr}} = \frac{E}{\lambda_0 \lambda_s^2}.$$

For visible light, taking $\lambda_0 \sim \lambda_s \sim 7000$ Å, we obtain $\rho_{\text{cr}} \sim 3$–5 atm for $E \sim 1$ MeV. In the case where a directed radiation flux is focused in a region of the order of several wavelengths—which in principle is possible for a very narrow line —there is a wide range of angular values. For a radiation density $\rho \sim 10$ atm, we find that an electron of energy 1 MeV is accelerated over a distance of the order of the wavelength $\lambda_0 \sim 7000$ Å by ~ 1 eV. If in the region of Cherenkov angles the radiation density depends sharply on the angle and decreases with increasing angle

$$\left(\frac{\theta}{N} \left| \frac{dN}{d\theta} \right| \gg 1, \ \frac{dN}{d\theta} < 0 \right),$$

then, as follows from (1), acceleration by radiation may be replaced by braking.

If the Cherenkov condition in the medium is satisfied only in the radio range and the radiation belongs to the radio range, then the critical radiation density is small. Thus, for $\lambda_0 \sim \lambda_s \sim 1$ cm, ρ_{cr} for ions ($E \sim 10^9$ eV) is only $\sim 10^{-9}$ atm. We note that supernova explosions, which according to present ideas(5) are sources of cosmic rays in the Galaxy, are accompanied by powerful radiation fluxes. Since on cosmic scales the distances traversed by particles are very large, the acceleration mechanism considered here may possibly play a role in the origin of cosmic rays.

4. The condition for absorption of Cherenkov radiation differs somewhat from the condition for emission of Cherenkov radiation. The first of them is satisfied, and the second is not, if

$$-\frac{\omega(n+1)}{2En} < \frac{m^2}{2E^2}\frac{n}{n-1} - 1 < \frac{\omega(n+1)}{2En}. \tag{4}$$

Here $E \gg m$ has been assumed. Let the radiation be present only in a narrow frequency interval near the frequency $\omega^{(0)}$, for which $n(\omega^{(0)}) = 1$. Put $\omega = \omega^{(0)} +$

* This holds only in the approximation e^2.

** It is assumed that the frequency ω_0 is far from the intrinsic frequencies of the medium (the medium is transparent for ω_0) and the nonlinear change of the electromagnetic properties of the medium may be neglected.

$+\delta\omega$. Let $\delta\omega = \delta\omega^{(0)}$ when $\frac{m^2}{2E^2}\frac{n}{n-1} = 1$. We have

$$\delta\omega^{(0)} \approx \frac{m^2}{2E^2}\left[\frac{\partial n\left(\omega^{(0)}\right)}{\partial\omega^{(0)}}\right]^{-}.$$

Condition (4) is satisfied in the frequency range

$$-\frac{\omega^{(0)}}{E} \ll \frac{\delta\omega - \delta\omega^{(0)}}{\delta\omega^{(0)}} < \frac{\omega^{(0)}}{E}, \tag{5}$$

and when the first of inequalities (5) is satisfied there is neither absorption nor emission. As the energy increases, the mean frequency $\omega^{(0)} + \delta\omega^{(0)}$ tends to $\omega^{(0)}$, and the frequency interval narrows. If the spectral line covers frequencies from $\omega^{(0)}$ to $\omega^{(0)} + \delta\omega^{(0)}$, where $\delta\omega^{(0)}$ corresponds to the initial energy of the particle, then, as the energy increases, the frequency region (5) will always lie inside the line, and the particle will only absorb radiation. From (1) we obtain:

$$\frac{dW}{dx} \sim 2\pi\lambda_0 r_0 \rho \left(\frac{mc^2}{E}\right)^3. \tag{6}$$

Here, for the estimate, it has been assumed that the line width $\Delta\omega$ is approximately equal to $\delta\omega^{(0)}$. Formula (6) gives the same order of magnitude as (3), since $1 - \frac{1}{n^2} \sim \frac{m^2}{E^2}$.

5. The above comparison of the mean fluctuation force acting on a particle from the radiation corresponds to the most favorable case, when other types of energy losses—in particular, ionization losses—are small (motion in a narrow channel). In comparison with the total losses, the critical radiation densities obtained must be increased by a factor equal to the ratio of ionization losses to Cherenkov losses.

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Note: Figure translations are in progress. See original paper for figures.