T, S-CURVES AND THE VERTICAL STABILITY OF OCEAN WATERS

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Abstract

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GEOPHYSICS

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T, S-CURVES AND THE VERTICAL STABILITY OF OCEAN WATERS

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As is known, the T, S-curves of oceanographic stations, or the curves $T = f(S)$ with respect to the parameter z, constructed in Helland-Hansen T, S-coordinates (Fig. 1), are an important diagnostic means for identifying water masses and studying their mutual transformation ([1]). An equally important characteristic of the static (and dynamic) state of stratified ocean waters is vertical stability ([2]):

$$E = \frac{g}{\rho}\frac{\delta\rho}{dz} = \frac{g}{\rho}\left[\frac{\partial\rho}{\partial T}\left(\frac{dT}{dz} - \frac{d\tau}{dz}\right) + \frac{\partial\rho}{\partial S}\frac{dS}{dz}\right], \tag{1}$$

usually computed as

$$E' = \frac{\partial\rho}{\partial T}\left(\frac{dT}{dz} - \frac{d\tau}{dz}\right) + \frac{\partial\rho}{\partial S}\frac{dS}{dz}. \tag{2}$$

Here T is temperature, S is salinity, ρ is density, z is depth; $d\rho/dT$ and $d\rho/dS$ are partial derivatives of the equation of state of seawater

$$\rho = \rho(T, S, p) \tag{3}$$

according to Knudsen and Ekman; finally, g is the acceleration of gravity and $d\tau/dz$ is the vertical adiabatic temperature gradient.

Since

$$\frac{\partial\rho}{\partial T} = f_1(T, S, p); \qquad \frac{\partial\rho}{\partial S} = f_2(T, S, p), \tag{4}$$

these derivatives are computed as

$$\left(\frac{\partial\rho}{\partial T}\right)_{STp} = \left(\frac{\partial\rho}{\partial T}\right)_{ST0} + \left[\Delta\left(\frac{\partial\rho}{\partial T}\right)\right]_{Tp} + \left[\Delta\left(\frac{\partial\rho}{\partial T}\right)\right]_{STp},$$

Fig. 1

Figure 1: Fig. 1

$$\left(\frac{\partial\rho}{\partial S}\right)_{STp} = \left(\frac{\partial\rho}{\partial S}\right)_{ST0} + \left[\Delta\left(\frac{\partial\rho}{\partial S}\right)\right]_{Sp} + \left[\Delta\left(\frac{\partial\rho}{\partial S}\right)\right]_{STp}, \tag{5}$$

where the square brackets contain corrections to $(\partial\rho/\partial T)_{ST0}$ and $(\partial\rho/\partial S)_{ST0}$ for pressure.

Neglecting these corrections (i.e., excluding the influence of pressure p), as well as the small value of the adiabatic gradient $d\tau/dz$, we obtain an expression for the principal part of the stability, determined only by the vertical distribution of temperature and salinity:

$$E'_{ST0} = \left(\frac{\partial\rho}{\partial T}\right)_{ST0}\frac{dT}{dz} + \left(\frac{\partial\rho}{\partial S}\right)_{ST0}\frac{dS}{dz}. \tag{6}$$

In oceanographic practice, T, S-curves and the stability criterion E are usually considered separately, and it is assumed that these two characteristics mutually complement one another in the analysis of ocean water masses ([3]). Meanwhile, these two characteristics are connected by a mathematical dependence, to the derivation of which this note is devoted.

Consider the value of the curvilinear integral along the T, S-curve (the curve $T = f(S)$):

$$I = \int_{(T_0,S_0)}^{(T,S)} P\,dS + Q\,dT. \tag{7}$$

In the case of the equation of state of seawater at zero (atmospheric) pressure

$$\rho = \rho(T, S) \tag{8}$$

the total differential $d\rho$ is written in the form

$$d\rho = \frac{\partial\rho}{\partial S}\,dS + \frac{\partial\rho}{\partial T}\,dT. \tag{9}$$

Fig. 1

In this case the integrand expression in (7) will be the total differential (9), with

$$P(T, S) = \frac{\partial\rho}{\partial S}, \qquad Q(T, S) = \frac{\partial\rho}{\partial T}, \tag{10}$$

and the condition

$$\frac{\partial P}{\partial T} \equiv \frac{\partial Q}{\partial S} \tag{11}$$

is identically satisfied.

Consequently, the Leibniz-Newton formula extended to a curvilinear integral is valid,

$$\rho(T,S) - \rho(T_0,S_0) = \int_{(T_0,S_0)}^{(T,S)} P\,dS + Q\,dT. \tag{12}$$

Further, the curvilinear integral

$$I = \int_{(T_0,S_0)}^{(T,S)} P(T,S)\,dS + Q(T,S)\,dT \tag{13}$$

can be computed with respect to the parameter z, taking into account that

$$S = \varphi(z), \qquad T = \psi(z), \tag{14}$$

namely, by the known formula:

$$I = \int_a^b \left[P(\varphi(z),\psi(z)) \cdot \varphi'(z) + Q(\varphi(z),\psi(z)) \cdot \psi'(z)\right] dz, \tag{15}$$

where $z = a$ at the point (T_0, S_0), and $z = b$ at the point (T, S) (Fig. 1).

In this case we obtain

$$I = \int_a^b \left(\frac{\partial \rho}{\partial S}\frac{dS}{dz} + \frac{\partial \rho}{\partial T}\frac{dT}{dz}\right) dz, \tag{16}$$

or

$$I = \int_a^b E'_{ST0}(z)\,dz. \tag{17}$$

The curvilinear integral along the T,S-curve from the point $a(T_0,S_0)$ to the point $b(T,S)$ is equal to the definite integral of the principal part of the stability between $z = a$ and $z = b$.

The quantity

$$\frac{g}{\rho} I = g\,\frac{\rho(b) - \rho(a)}{\rho}$$

is equal to the reduced acceleration of gravity and characterizes the reserve of potential energy of a layer of thickness from $z = a$ to $z = b$.

Since the integrand in (7) is a total differential, the value of the curvilinear integral I does not depend on the path of integration, but is determined only by the positions of the endpoints a and b (Fig. 1).

Let us note in conclusion that condition (11) makes it possible to write the equations

$$\frac{\partial \rho}{\partial S} = \frac{\partial \gamma}{\partial T}; \qquad \frac{\partial \rho}{\partial T} = -\frac{\partial \gamma}{\partial S}, \tag{18}$$

formally analogous to the Cauchy–Riemann equations; in equations (18) the function ρ is formally analogous to the stream function ψ, and the function γ to the velocity potential φ in the Cauchy–Riemann equations. The function γ may be called the gravitational potential in a stratified fluid, and the function ρ, the function of isopycnicity. The families of isolines $\rho = \text{const}$ and $\gamma = \text{const}$ on the T, S-diagram are orthogonal.

Equations (18) open up the possibility (admittedly a limited one, since the two indicated families of isolines on the T, S-diagram are fixed) for the study of the equation of state of seawater and of the processes of mixing and transformation of water masses (including processes of horizontal isopycnic transformation) within the framework of the theory of functions of a complex variable.

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Note: Figure translations are in progress. See original paper for figures.