

ON STABILITY IN THE LARGE OF NONLINEAR PULSE AUTOMATIC SYSTEMS

![Fig. 1](image)

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Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Abstract

Full Text

CYBERNETICS AND CONTROL THEORY

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ON STABILITY IN THE LARGE OF NONLINEAR PULSE AUTOMATIC SYSTEMS

(Presented by Academician B. N. Petrov, 30 I 1962)

Nonlinear pulse automatic systems (PAS), including digital automatic systems (DAS), play an important role in modern technology. For the study of stability in the large of nonlinear PAS, Lyapunov's direct method, extended to difference equations, has recently been brought to bear ⁽¹⁾.

Fig. 1

In the present work it is shown that this problem can be solved much more simply and effectively if one uses the approach of V. M. Popov, proposed for the study of stability in the large of continuous automatic systems ⁽²⁾. This approach is closely connected with the familiar concept of the "frequency characteristic," frees one from arbitrariness in the choice of a quadratic form, as occurs in Lyapunov's direct method, and, finally, gives the broadest sufficient conditions for stability in the large.

Fig. 2

Consider a nonlinear PAS with amplitude modulation (Fig. 1). Suppose that the nonlinear characteristic of the pulse element $\Phi(x)$ (Fig. 2) satisfies the following conditions:

$$\text{a) } \Phi(0) = 0; \quad \text{b) } \Phi(x[n])/x[n] < k. \quad (1)$$

The reduced continuous part is assumed to be stable, i.e., the poles of its transfer function have negative real parts.

In this case the initial conditions can be represented in the form of an action applied to the input of the impulse element $f[n]$, with

$$\lim_{n \rightarrow \infty} f[n] = 0. \quad (2)$$

The nonlinear IAS will be stable, and moreover asymptotically stable, if

$$\lim_{n \rightarrow \infty} x[n] = 0. \quad (3)$$

The equation of the nonlinear IAS with respect to the error can be represented in the form ⁽³⁾

$$x[n] = f[n] - \sum_{m=0}^n w[n-m, 0] \Phi(x[m]), \quad (4)$$

where

$$w[n] = w(\bar{t})_{\bar{t}=n} \quad (5)$$

is the impulse response of the reduced continuous part, satisfying the condition

$$\lim_{n \rightarrow \infty} w[n] = 0. \quad (6)$$

By analogy with the approach of V. M. Popov ⁽²⁾, introduce the auxiliary functions

$$\varphi_N[n] = \begin{cases} \Phi(x[n]), & 0 \leq n \leq N, \\ 0, & n < 0, n > N; \end{cases} \quad (7)$$

$$\psi_N[n] = x_N[n] - \frac{1}{k} \varphi_N[n], \quad (8)$$

where

$$x_N[n] = f[n] - \sum_{m=0}^n w[n-m, 0] \varphi_N[m]. \quad (9)$$

It is obvious that for $0 \leq n \leq N$ the quantity $x_N[n]$ coincides with $x[n]$ of equation (4).

Form the expression

$$\rho_N = \sum_{n=0}^{\infty} \varphi_N[n] \psi_N[n] = \sum_{n=0}^N \Phi(x[n]) \psi_N[n]. \quad (10)$$

According to the Lyapunov-Parseval theorem (3),

$$\rho_N = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_N^*(-j\bar{\omega}) \Psi_N^*(j\bar{\omega}) d\bar{\omega}, \quad (11)$$

where

$$\Phi_N^*(j\bar{\omega}) = \sum_{n=0}^{\infty} e^{-j\bar{\omega}n} \varphi_N[n]; \quad \Psi_N^*(j\bar{\omega}) = \sum_{n=0}^{\infty} e^{-j\bar{\omega}n} \psi_N[n] \quad (12)$$

are spectral functions which, by virtue of conditions (2), (6), (7), exist.

Computing the spectral functions by formulas (12), taking (7)–(9) into account, and substituting them into (11), after transformations we obtain

$$\rho_N = -\frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \sqrt{\operatorname{Re} \Pi^*(j\bar{\omega})} \Phi_N^*(j\bar{\omega}) - \frac{F^*(j\bar{\omega})}{2\sqrt{\operatorname{Re} \Pi^*(j\bar{\omega})}} \right|^2 d\bar{\omega} + \frac{1}{8\pi} \int_{-\pi}^{\pi} \frac{|F^*(j\bar{\omega})|^2}{\operatorname{Re} \Pi^*(j\bar{\omega})} d\bar{\omega}, \quad (13)$$

where

$$\Pi^*(j\bar{\omega}) = W^*(j\bar{\omega}) + \frac{1}{k} > 0. \quad (14)$$

In formulas (13) and (14), $F^*(j\bar{\omega})$ is the spectral function corresponding to $f[n]$, and $W^*(j\bar{\omega})$ is the frequency characteristic of the open-loop linear IAS corresponding to $w[n, 0]$. Since the first integral in (13) is negative, by discarding it we obtain the inequality

$$\rho_N \leq \frac{1}{8\pi} \int_{-\pi}^{\pi} \frac{|F^*(j\bar{\omega})|^2}{\operatorname{Re} \Pi^*(j\bar{\omega})} d\bar{\omega} = C, \quad (15)$$

where, according to (14), $C > 0$. Note that the right-hand side of the inequality does not depend on N .

Substituting now into (10) the values $\psi_N[n]$ from (8), we represent inequality (15) in the form

$$\rho_N = \sum_{n=0}^N \Phi(x[n]) x[n] \left(1 - \frac{\Phi(x[n])}{kx[n]} \right) < C. \quad (16)$$

It is valid for any N .

The sum entering (16), by virtue of conditions (1), is positive, and, consequently, this sum is bounded. From the boundedness of a sum containing positive terms

Fig. 3

Figure 3: Fig. 3

(by virtue of property (16)), according to the theorem on convergence of series with positive terms ⁽⁴⁾, we conclude that

$$\lim_{n \rightarrow \infty} \Phi(x[n]) x[n] \left(1 - \frac{\Phi(x[n])}{kx[n]} \right) = 0, \quad (17)$$

whence it follows that

$$\lim_{n \rightarrow \infty} x[n] = 0. \quad (18)$$

Thus, we arrive at an analogue of the theorem of V. M. Popov ⁽²⁾ for nonlinear pulse systems:

A nonlinear IAS possessing a characteristic $\Phi(x)$ satisfying conditions (1), and having a stable continuous part, will be globally stable if

$$\operatorname{Re} \Pi^*(j\bar{\omega}) > 0, \quad 0 \leq \bar{\omega} \leq \pi, \quad (19)$$

i.e., if

$$\operatorname{Re} W^*(j\bar{\omega}) > -\frac{1}{k}. \quad (20)$$

This theorem is also valid for nonlinear IAS containing a neutral continuous part whose transfer function has one pole at zero, while the remaining poles have negative real part; only in this case inequality (16) is replaced by $a < \Phi(x[n])/x[n] < k$,

where α is a sufficiently small quantity (Fig. 2). Fulfillment of sufficient conditions for stability in the large can be established by means of the geometric stability criterion. For this it is sufficient, in the plane of the frequency characteristic,

$$W^*(j\bar{\omega}) = \operatorname{Re} W^*(j\bar{\omega}) + j \operatorname{Im} W^*(j\bar{\omega}) \quad (21)$$

to draw a vertical straight line to the frequency characteristic in such a way that the latter lies entirely to the right (Fig. 3). This straight line cuts off a segment of length $1/k$, whence the value of k is obtained that determines the opening of the sector (Fig. 2) within which the nonlinear characteristic of the pulse element is located.

Fig. 3

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