PHYSICS

V. P. GLOTOV

1962

SovietRxiv

Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

PHYSICS

V. P. GLOTOV

ON COHERENT SCATTERING OF PLANE AND SPHERICAL WAVES IN DEEP-WATER LAYERS CONTAINING DISCRETE INHOMOGENEITIES

(Presented by Academician N. N. Andreev, 16 V 1961)

It is usually assumed that the intensity of sound scattering in marine bodies of water (layer reverberation) is the sum of the intensities arriving at the receiving point from each scatterer contained in the insonified volume ([1,2]). Meanwhile, it is obvious that if the mean distances between scatterers are comparable with the wavelength of the radiation, then interference of the sound scattered by discrete inhomogeneities may be of substantial importance. With a sufficiently large concentration of scatterers in the layer, the interference part (coherent scattering) predominates over the noninterference part (incoherent scattering). Using the relation between the interference and noninterference parts of the scattered intensity, one can obtain additional information on the structure of the layer.

Fig. 1

1. Scattering of a plane wave. Consider a horizontally situated layer, infinite in extent along x and y and of thickness H (Fig. 1). Suppose that in the layer scatterers of arbitrary dimensions are distributed chaotically but, on the average, uniformly. We shall denote the mean number of scatterers per unit volume by $\overline{N}$.

Let a plane wave of unit amplitude be incident on the layer along the z-axis,

$e^{i\bar{k}z}$, where $\bar{k} = k + ik_1$ is the wave number. Suppose further that in the layer there are scatterers of "water-like" structure, i.e., scattering sound so weakly that their amplitude influence on one another through scattered waves is negligibly small.* The wave scattered by an individual scatterer may be represented in the form

$$A\,\frac{e^{i\bar{k}(z+r)}}{r}, \tag{1}$$

where A is the amplitude of elementary scattering, depending on the physical nature of the scatterer and, in addition, on its orientation in space if the elementary scattering is directional; r is the distance from the center of the scatterer to the receiving point.

We shall find the scattered field at a certain point on the axis $z = 0$, situated at an arbitrary distance from the layer. Suppose that the concentration of scatterers—

* By scatterers of "water-like" structure we mean scatterers of biological nature (fish, jellyfish, plankton) and other suspended matter, whose refractive index μ differs little from unity and satisfies the inequality $|\mu| \ll (1 + ka)/ka$, where a is the mean size of the inhomogeneity.

the scatterers in the layer is sufficiently large, so that in the volumetric Fresnel zone there is at least one scatterer, i.e., the condition

$$\overline{N}\lambda^2 z_0 \gg 1 \quad \left(\lambda = \frac{2\pi}{k}\right). \tag{2}$$

is satisfied.

Denote by N_n the number of scatterers in some volume w_n, small in comparison with the volumetric Fresnel zone ($w_n < \lambda^2 z_0$). Because of fluctuations in the number of scatterers per unit volume, the quantity N_n, naturally, differs from the mean value $\overline{N}w_n$. The elementary waves coming from all the scatterers located in the volume w_n arrive at the point of reception with one and the same phase. Therefore the total field from all scatterers located in the volume with number n will be

$$P_n = B_n \frac{e^{ikR_n}}{r_n}, \tag{3}$$

where B_n is the sum of the amplitudes of the different scatterers located in the volume w_n; $kR_n = k(z_n + r_n)$ is the phase of the wave scattered by the same volume; z_n and r_n are the coordinate and the distance to the reception point of the center of w_n.

Let us find the mean (statistical) value of the intensity of the scattering arriving at the reception point $z = 0$. We have

$$\overline{I} = \overline{P_{nP} m^*} = \sum_n \sum_m \frac{1}{r_n r_m}\, e^{ik(R_n - R_m)} e^{-k_1(R_n + R_m)} \overline{(B_{nB} m)}. \tag{4}$$

Averaging is carried out over all possible arrangements of scatterers in the layer of thickness H. The averaging in the right-hand side of (4) pertains only to the product $(B_{nB}m)$, since the auxiliary volumes w_n and the phase relations do not change as a result of fluctuations in the number of scatterers in these volumes.

We have:

$$\overline{I} = \sum_n \sum_m \frac{1}{r_n r_m} e^{ik(R_n - R_m) - k_1(R_n + R_m)} \overline{(B_{nB}m)} =$$

$$= \left| \sum_n \frac{\overline{B}_n}{r_n} e^{ikR_n - k_1 R_n} \right|^2 + \sum_n \frac{1}{r_n} e^{ikR_n - k_1 R_n} \overline{(\delta B_n)^2} +$$

$$+ \sum_{n \neq m} \sum \frac{1}{r_n r_m} e^{ik(R_n - R_m) - k_1(R_n + R_m)} \overline{(\delta B_n \cdot \delta B_m)}, \tag{5}$$

where $\overline{\delta B_n} = \overline{\delta B_m} = 0$.

Assuming further that fluctuations in the number of scatterers with one and the same amplitude in different volumes are uncorrelated, and that fluctuations of scatterers of different amplitude in one and the same volume are likewise uncorrelated, we obtain

$$\overline{I} = \overline{N}^2 \overline{A}^2 \left| \sum_n \frac{1}{r_n} e^{ikR_n - 2k_1 R_n} w_n \right|^2 + \overline{N}\, \overline{A^2} \sum_n \frac{1}{r_n^2} e^{-2k_1 R_n} w_n, \tag{6}$$

where $\overline{A}^2$ is the square of the mean amplitude of the scatterers in the layer; $\overline{A^2}$ is the mean square of the amplitude of the scatterers in the layer.

The sums in (6) may be replaced by integrals over the volume of the layer. Taking into account that $R = z + r$, $dw = 2\pi r\, dr\, dz$, we shall have:

$$\overline{I} = \overline{N}^2 \overline{A}^2 |I_1|^2 + \overline{N}\, \overline{A^2} I_2, \tag{7}$$

where

$$I_1 = 2\pi \int_{z_0}^{z_0 + H} e^{ikz - k_1 z}\, dz \cdot \int_z^\infty e^{ikr - k_1 r}\, dr, \qquad I_2 = 2\pi \int_{z_0}^{z_0 + H} e^{-2k_1 z}\, dz \cdot \int_z^\infty e^{-2k_1 z}\, dr. \tag{8}$$

The first term in (7) gives a coherent contribution to the scattered intensity, and the second an incoherent contribution. In the coherent part the scattering is added in amplitude, with allowance for the phase of the scatterers; in the incoherent part the scattering is added in energy. Calculating the integrals in (8) for small sound absorption in the layer and outside it, we obtain:

$$I_1 = 2\pi i \frac{e^{ik(2z_0+H)}}{k^2} \sin kH, \qquad I_2 = 2\pi H(-\ln k_1\gamma z_0), \qquad \gamma = 1.781 \quad (9)$$

$$(2k_1 z_0 \ll 1, \quad 2k_1 H \ll 1, \quad k_1 \ll k, \quad H \ll z_0).$$

For the ratio of the coherent part of the scattered intensity to its incoherent part, taking (7) and (9) into account, we obtain

$$\frac{\overline{I}_{\text{coh}}}{\overline{I}_{\text{incoh}}} = 2\pi\overline{N}\,\frac{\overline{A}^2}{\overline{A^2}}\,\frac{\sin^2 kH}{k^4 H(-\ln 2k_1\gamma z_0)}. \qquad (10)$$

It follows from formula (10) that, for small sound absorption in the medium, coherent scattering is filtered by the layer. By varying in this way the wavelength of the radiation ($\lambda = 2\pi/k$) and the position of the receiving point ($z = z_0$) and by measuring the ratio $I_{\text{coh}}/I_{\text{incoh}}$ (see below), one can estimate the mean concentration and the mean squares of the amplitudes of the scatterers in the layer.

2. Scattering of a spherical wave. Let a spherical wave $e^{i\bar{k}r}/r$ be emitted from the point $z = 0$ (Fig. 1), where r is the distance from the point of emission, $\bar{k} = k + ik_1$ is the wave number. It is required to find the scattered field at a point on the z-axis, coinciding with the point of emission.

The wave scattered by an individual scatterer located at a large distance from the emission-reception point ($r \gg 2\pi a_m^2/\lambda$, where a_m is the maximum size of the scatterer) can be represented in the form

$$D\,\frac{e^{2i\bar{k}r}}{r^2}, \qquad (11)$$

where D is the amplitude of elementary scattering, depending on the physical nature of the scatterer and its orientation in space; r is the distance from the center of the scatterer to the emission-reception point.

Making the same assumptions concerning the concentration and fluctuations in the number of scatterers in the auxiliary volumes w_n as in the case of a plane wave, and carrying out transformations analogous to (4), (5), and (6), we obtain

$$\overline{I} = \overline{N}^2\overline{D}^2|I_3|^2 + \overline{N}\,\overline{D^2}\,I_4, \qquad (12)$$

where

$$I_3 = 2\pi \int_{z_0}^{z_0+H} dz \int_z^\infty \frac{e^{2ikr-2k_1 r}}{r}\, dr, \qquad I_4 = 2\pi \int_{z_0}^{z_0+H} dz \int_z^\infty \frac{e^{-4k_1 r}}{r^3}\, dr. \tag{13}$$

Calculating the integrals in (13) for small sound absorption in the layer and outside it, we obtain

$$I_3 = -\frac{\pi i e^{ik(2z_0+H)}}{k^2 z_0} \sin kH, \qquad I_4 = \pi \frac{H}{z_0^2}$$

$$(4k_1 H \ll 1, \quad 4k_1 z_0 \ll 1, \quad kz_0 \gg 1). \tag{14}$$

For the ratio of the coherent part of the scattered intensity to its incoherent part we obtain, taking (12), (14) into account:

$$\frac{\overline{I}_{\text{coh}}}{\overline{I}_{\text{incoh}}} = \overline{N}\, \frac{\pi}{k^4 H}\, \frac{\overline{D}^2}{\overline{D^2}} \sin^2 kH. \tag{15}$$

Let us consider a concrete example. Suppose that in a layer of thickness $H = 2.25\lambda$ there are identical scatterers, 10 scatterers per 1 liter of water. In

$\lambda = 1$ m, according to formula (15) ($\overline{D^2} = \overline{D}^2$), we obtain $I_{\text{coh}}/I_{\text{incoh}} \sim 10$, i.e., the layer practically reflects, rather than scatters, the spherical wave. At a lower concentration of scatterers ($\overline{N} \ll 1$ scatterer per 1 l), on the contrary, the layer scatters the same wave.

It is curious that the physical nature of the scatterers does not affect the final result (formulas (10) and (15)). Some influence is exerted by the statistical distribution function of the amplitudes of the scatterers in the layer, which depends, in turn, on the sizes of the scatterers and, moreover, on their orientation in space, if the elementary scattering is directional.

3. Fluctuations of the sound scattered by the layer. Owing to the random distribution of scatterers in the layer, the scattering amplitude will fluctuate in time about its mean value. If the concentration is small ($I_{\text{incoh}} \gg I_{\text{coh}}$), then the phases of the scattered waves arriving at the reception point are random, and the statistical distribution of scattering fluctuations will obey the Rayleigh law, for which the relative variance of the fluctuations tends to a constant value equal to 0.27. In the opposite case, when the concentration is large ($I_{\text{incoh}} \ll I_{\text{coh}}$), the scattering from the layer has a regular character, i.e., reflection from the layer occurs with constant amplitude and phase. The relative variance of the fluctuations in this case tends to zero.

In the intermediate case, for any ratio of I_{coh} and I_{incoh}, the statistical distribution of fluctuations will obey the law (3)

$$V(P) = \frac{2P}{\sum P_s^2} \exp\left[-\frac{P_0^2 + P^2}{\sum P_s^2}\right] I_0\left(\frac{2PP_0}{\sum P_s^2}\right), \tag{16}$$

where $V(P)$ is the differential distribution function of the scattering amplitudes arriving at the reception point; P_s are the random amplitudes of the (incoherent) elementary scatterers with random phases; P_0 is the regular (coherent) scattering amplitude with constant phase; $I_0(z)$ is the modified Bessel function of zero order.

Calculating, with the aid of (16), the relative variance $\chi = \overline{P^2}/\overline{P}^2 - 1$ and expressing it as a function of $P_0^2/\sum \overline{P_s^2} = I_{\text{coh}}/I_{\text{incoh}} = \beta^2$, we obtain

$$\chi + 1 = \frac{4}{\pi} \frac{e^{\beta^2}(1+\beta^2)}{\left[(1+\beta^2)I_0(\beta^2/2) + \beta^2 I_1(\beta^2/2)\right]^2}. \tag{17}$$

The relative variance χ can be obtained experimentally, namely:

$$\chi + 1 = \left[\frac{1}{M}\sum_{i=1}^{M} P_i^2\right] \Bigg/ \left(\frac{1}{M}\sum_{i=1}^{N} P_i\right)^2, \tag{18}$$

where M is the number of scattering amplitudes P_i taken for processing. By measuring the relative variance of the fluctuations of scattering from layers, one can determine from formula (17) the quantity $\beta^2 = I_{\text{coh}}/I_{\text{incoh}}$, and then, from formulas (10) or (15), estimate all parameters of the scattering layer.

The author expresses deep gratitude to Prof. Yu. M. Sukharevsky for his attention to the work and for useful discussion of the results.

Acoustics Institute
Academy of Sciences of the USSR

Received
16 V 1961

References

1. Yu. M. Sukharevsky, DAN, **55**, No. 9, 825 (1947).

2. S. Machlup, J. B. Hersey, Deep-Sea Res., **3**, No. 1 (1955).

3. V. I. Bunimovich, *Fluctuation Processes in Radio Receiving Devices*, 1951.

Note: Figure translations are in progress. See original paper for figures.