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Abstract

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PHYSICS

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RADIATIVE CORRECTIONS TO THE INTENSITY OF CHERENKOV RADIATION OF CHARGED PARTICLES

(Presented by Academician V. I. Veksler, 27 XII 1961)

1. The classical theory of Cherenkov radiation, developed by I. E. Tamm and I. M. Frank ([1],[2]) and E. Fermi ([3]), is known to describe the macroscopic losses of charged particles in first order in e^2. Quantum corrections ([4]), which take into account recoil in radiation in the same order in e^2, are, generally speaking, small.* In the first approximation these corrections have relative order ω/ε_p, where ω is the frequency of the emitted quantum, and ε_p is the energy of the particle ($\hbar = c = 1$). Here we shall consider radiative corrections to the energy losses, i.e., we shall take into account terms of the next order in e^2 (more precisely, terms e^4 and $e^4 \ln e^2$).

For the calculation we shall use the Green-function method ([5],[6]), which makes it possible to find energy losses from the poles of the analytic continuation of the particle Green function (see ([5])). The calculations are considerably simplified if, in all orders in e^2, only terms of first order in ω/ε_p are retained (more precisely, if terms of order ω/ε_p are neglected in comparison with unity). For the particle Green function G in an isotropic homogeneous medium we then obtain the equation

$$[E - \varepsilon_p - \Sigma(E, \mathbf{p})]G = 1, \tag{1}$$

where E is the particle energy; $\mathbf{p}$ is its momentum ($\varepsilon_p = \sqrt{\mathbf{p}^2 + m^2}$ is the energy of a free particle); $\Sigma(E, \mathbf{p})$ is the mass operator, determined by the usual graphical correspondence rules with the following rules:

1) to an electron internal line one assigns the operator

$$G_0(E, \mathbf{p}) = (E - \varepsilon_p + i\delta)^{-1}, \qquad \delta \to +0;$$

2) to a photon line (for simplicity, at temperature $T = 0$) one assigns

$$\frac{2e^2}{(2\pi)^4}\left\{\operatorname{Im} D_r^t(\omega,\mathbf{k})\cdot\frac{[\mathbf{vk}]^2}{\mathbf{k}^2} - \operatorname{Im} D_r^l(\omega,\mathbf{k})\right\}, \tag{2}$$

$$D_r^t(\omega,\mathbf{k}) = \frac{4\pi}{\mathbf{k}^2-\omega^2\varepsilon^t(\omega,\mathbf{k})}, \qquad D_r^l(\omega,\mathbf{k}) = \frac{4\pi}{\mathbf{k}^2\varepsilon^l(\omega,\mathbf{k})}, \qquad \mathbf{v} = \frac{\mathbf{p}}{\varepsilon_p};$$

3) the energy E and momentum $\mathbf{p}$ of an electron line must be chosen in accordance with the conservation laws at the vertices. Integration is performed over all frequencies ω of the quanta from 0 to ∞, and over momenta $\mathbf{k}$ from $-\infty$ to $+\infty$.**

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* An exception may be the emission of a longitudinal quantum (plasmon) near the energy threshold ([5]).

** These rules may be justified by using the equation for the Green function of a spin-$\frac{1}{2}$ particle. In view of $\omega/\varepsilon_p \ll 1$, in the Green function of a free particle one may neglect the positron part, which is proportional (for $\omega/\varepsilon_p \ll 1$) to ε_p^{-1}, whereas the electron part is proportional to ω^{-1}. Then

$$G_0(E,\mathbf{p}) = \Lambda_p^-(\varepsilon_p - E - i\delta)^{-1}, \qquad \delta\to+0, \qquad \Lambda_p^- = (2\varepsilon_p)^{-1}(m - i\hat{p}),$$

In deriving the rules presented above, the causal Green function $D_c(\omega,\mathbf{k})$ was expressed, with the aid of dispersion relations, in terms of the imaginary part of the retarded $D_r(\omega,\mathbf{k})$. The latter, in turn, is expressed[5] through the longitudinal $\varepsilon^l(\omega,\mathbf{k})$ and transverse $\varepsilon^t(\omega,\mathbf{k})$ dielectric constants, which, for the simplest media, can be calculated by perturbation theory[7,8], or else specified phenomenologically.

The effective energy spectrum $E = E(\mathbf{p})$ is determined from the equation

$$E - \varepsilon_p - \Sigma(E,\mathbf{p}) = 0. \tag{3}$$

The imaginary part of E describes the energy losses of the particle.

2. In first order in e^2, the mass operator has the form (Fig. 1).

$$\Sigma_1 = \frac{2e^2}{(2\pi)^4}\int_0^\infty d\omega\int d\mathbf{k}\,(E-\varepsilon_{\mathbf{p}-\mathbf{k}}-\omega)^{-1}\left\{\frac{[\mathbf{vk}]^2}{k^2}\operatorname{Im} D_r^t(\omega,\mathbf{k}) - \operatorname{Im} D_r^l(\omega,\mathbf{k})\right\}. \tag{4}$$

It is easy to see that, in the first approximation for the imaginary part, putting in (4) $E = \varepsilon_p + i\delta$ and $\varepsilon_p - \varepsilon_{\mathbf{p}-\mathbf{k}} \simeq \mathbf{kv} + i\delta$, and replacing the energy denominator

Fig. 1 and Fig. 2: schematic mass-operator diagrams

Figure 1: Fig. 1 and Fig. 2: schematic mass-operator diagrams

by $-i\pi\delta(\omega-\mathbf{kv})$, we obtain the well-known classical result for media with spatial dispersion[5–7].

Fig. 1 Fig. 2

For obtaining the radiative corrections it is necessary both to calculate Σ_1 more accurately and to take into account the mass operator in the next order in e^2 (Fig. 2).

$$\Sigma_2 = \frac{4e^4}{(2\pi)^8}\int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \int d\mathbf{k}_1\, d\mathbf{k}_2 \left\{\frac{[\mathbf{vk}_1]^2}{k_1^2}\operatorname{Im} D_r^t(\omega_1,\mathbf{k}_1) - \operatorname{Im} D_r^l(\omega_1,\mathbf{k}_1)\right\}$$
$$\times \left\{\frac{[\mathbf{vk}_2]^2}{k_2^2}\operatorname{Im} D_r^t(\omega_2,\mathbf{k}_2) - \operatorname{Im} D_r^l(\omega_2,\mathbf{k}_2)\right\}(E-\varepsilon_{\mathbf{p}-\mathbf{k}_1}-\omega_1)^{-1}$$
$$\times (E-\varepsilon_{\mathbf{p}-\mathbf{k}_1-\mathbf{k}_2}-\omega_1-\omega_2)^{-1}\left[(E-\varepsilon_{\mathbf{p}-\mathbf{k}_2}-\omega_2)^{-1}+(E-\varepsilon_{\mathbf{p}-\mathbf{k}_1}-\omega_1)^{-1}\right]. \quad (5)$$

$$\Lambda_{\mathbf{p}}^-$$

is the projection operator onto positive energies. In constructing the mass operator with the aid of $G_0(E,\mathbf{p})$, one may replace $\Lambda_{\mathbf{p}-\mathbf{k}}^-$, owing to $k \ll p$, by $\Lambda_{\mathbf{p}}^-$. The expressions of the type $\gamma_i\Lambda_{\mathbf{p}}^-\gamma_j\Lambda_{\mathbf{p}}^-\gamma_s\Lambda_{\mathbf{p}}^-\gamma_k$ ... which occur in the mass operator can be transformed to a form containing $\Lambda_{\mathbf{p}}^+ = (2\varepsilon_p)^{-1}(m+i\hat{p})$ on the left, and all $\gamma_i\gamma_j$ ... on the right.

Then one can verify that, owing to the symmetry of the integrand with respect to the momenta of the quanta, the mass operator has the form $\Sigma = \delta m + i\gamma_\mu\delta p_\mu$, and therefore ($\delta p_4 = -i\delta E$)

$$E(\mathbf{p}) = \left[(\mathbf{p}+\delta\mathbf{p})^2 + (m+\delta m)^2\right]^{1/2} - \delta E \simeq \varepsilon_p + \varepsilon_p^{-1}\mathbf{p}\,\delta\mathbf{p} + \varepsilon_p^{-1} m\,\delta m - \delta E. \quad (*)$$

Taking into account terms quadratic in $\delta\mathbf{p}, \delta m, \delta E$ would mean taking into account terms of order ω/ε_p in comparison with 1. It is easy to verify from () *that terms proportional to* $\Lambda_{\mathbf{p}}^+$ *make no contribution to* $E(\mathbf{p})$. *In the remaining terms, the calculations by means of ()* lead to the result that all γ_i $(i=1,2,3)$ must be replaced by $v_i = p_i\varepsilon_p^{-1}$, and γ_4 by 1.

For renormalization it is sufficient to replace $D_r^t(\omega,\mathbf{k})$ by $D_r^t(\omega,\mathbf{k}) - D_{r,B}^t(\omega,\mathbf{k})$, where $D_{r,B}^t$ is D_r^t for $\varepsilon^t = 1$.*

In order to take correctly into account terms of the form $e^4 \ln e^2$, we shall solve the dispersion equation (3) by successive approximations. In the zeroth approximation we put $E = \varepsilon_p$. We substitute this value into Σ_1 and obtain $E = \varepsilon_p + \lambda_0 - i\gamma_0$. We substitute the value found into $\Sigma_1 - \Sigma_1|_{E=\varepsilon_p} + \Sigma_2$, and carry out the calculation in the first nonvanishing approximation, etc. This procedure gives, in the expansion of the effective energy spectrum, terms $e^{2\nu}\varphi_\nu(\ln e^2)$, successively decreasing with increasing ν (for small ν).

3. Let us consider transparent media and neglect spatial dispersion, $\varepsilon^t = \varepsilon^l = \varepsilon(\omega) = n^2(\omega)$:

$$\operatorname{Im} D_r^t(\omega, \mathbf{k}) = 4\pi^2 \delta\big(\mathbf{k}^2 - \omega^2 n^2(\omega)\big)\,, \qquad \operatorname{Im} D_r^l(\omega, \mathbf{k}) = -\frac{4\pi^2}{\mathbf{k}^2}\delta\big(n^2(\omega)\big)\,. \tag{6}$$

Calculation by the procedure given above yields

$$\gamma = -\operatorname{Im} E(\mathbf{p}) = \gamma_0 \left(1 - \frac{e^2}{\pi}\Delta^t\right), \qquad \Delta^t = \operatorname{Re}\int_{n^2>0}^{\infty} \Delta^t_\omega \, d\omega - \operatorname{Re}\int_0^\infty \Delta^t_{\omega,B}\, d\omega, \tag{7}$$

where

$$\Delta^t_\omega = \frac{v}{2(E_0 - \varepsilon_p)}\left(1 - \frac{1}{n^2 v^2}\right) \ln \frac{\omega - (E_0 - \varepsilon_p)(1 - nv)^{-1}}{\omega - (E_0 - \varepsilon_p)(1 + nv)^{-1}} +$$

$$+ \frac{3}{\omega n} + \frac{4\omega - 3(E_0 - \varepsilon_p)}{2\omega^2 n^2 v} \ln \frac{E_0 - \varepsilon_p - \omega(1 - nv)}{E_0 - \varepsilon_p - \omega(1 + nv)},$$

$$\Delta^t_{\omega,B} = \Delta^t_\omega\big|_{n(\omega)=1} \tag{8}$$

$E_0 - \varepsilon_p = \lambda_0 - i\gamma_0$ (λ_0 and γ_0 are the real and imaginary parts in the e^2 approximation).

It should be emphasized that the longitudinal part D^l, in the approximation under consideration (spatial dispersion neglected), does not contribute to Δ. For high frequencies,

$$\Delta^t_\omega = \frac{2}{\omega n}\left\{2 + \frac{1}{nv}\ln\left|\frac{1 - nv}{1 + nv}\right|\right\}, \qquad \Delta^t_{\omega,B} = \frac{2}{\omega}\left\{2 + \frac{1}{v}\ln\frac{1 - v}{1 + v}\right\}. \tag{9}$$

The energy losses of the particle in the medium, W, will be

$$W=\int_0^\infty 2\omega\gamma_\omega\,d\omega,\qquad \gamma=\int_0^\infty \gamma_\omega\,d\omega,\qquad \gamma_\omega=\gamma_{0,\omega}\left(1-\frac{e^2}{\pi}\Delta^t\right),\qquad \gamma_0=\gamma_0^t+\gamma_0^l. \tag{10}$$

γ_0^l and γ_0^t are the probabilities of emission of a longitudinal and a transverse quantum.

When spatial dispersion is neglected,

$$\gamma_{0,\omega}^t=e^2v\left(1-\frac{1}{n^2v^2}\right)\left(\frac{1}{2}+\frac{1}{2}\frac{nv-1}{|nv-1|}\right),\qquad \gamma_{0,\omega}^l=\frac{e^2}{v}\sum_s\left|\frac{\partial n^2(\omega_s)}{\partial\omega_s}\right|^{-1}\ln\frac{\chi_{\max}v}{\omega_s}, \tag{11}$$

where ω_s are the zeros of $n^2(\omega)$; $\chi_{\max}$ is the cutoff factor introduced in the usual way, disappearing after matching (for a plasma see [9]). An estimate of the magnitude of the radiative corrections is given by calculation for the simplest case $n^2=1-\omega_0^2/\omega^2$

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* The change in the mass of the particle in the medium, described by the real part of $E-\varepsilon_p$, is an observable effect and is simply related to the magnitude of the energy loss when the particle passes through a blurred interface of media [5]. The imaginary part of $E-\varepsilon_p$, describing the energy loss of the particle in a homogeneous medium, is naturally also an observable effect.

$$\Delta^t=\left(1+\ln\frac{\xi_0^2}{1-v^2}\right)\left(2+\frac{1}{v}\ln\frac{1-v}{1+v}\right)+\frac{1}{2v}\ln^2(1-v)-$$

$$-\frac{1}{2v}\ln^2(1+v)+\frac{1}{v}\ln(1+v)\ln(1-v)+$$

$$+4\ln 2+\frac{2}{v}\psi(1-v)+\frac{2}{v}\psi\left(\frac{1}{1+v}\right)-\frac{2}{v}\psi\left(\frac{1-v}{1+v}\right)+2-\frac{\pi^2}{3v}+$$

$$+\sqrt{1+\frac{1-v^2}{\xi_0^2}}\ln\frac{\sqrt{1+\frac{1-v^2}{\xi_0^2}}-1}{\sqrt{1+\frac{1-v^2}{\xi_0^2}}+1}+\frac{1}{2}\left[\ln\frac{\sqrt{1+\frac{1-v^2}{\xi_0^2}}-1}{\sqrt{1+\frac{1-v^2}{\xi_0^2}}+1}\right]^2, \tag{12}$$

where

$$\psi(x)=\sum_{k=1}^{\infty}x^k\frac{1}{k^2},$$

$$\xi_0 = \left|\frac{E_0 - \varepsilon_p}{\omega_0}\right| = \left|\frac{\lambda_0 - i\gamma_0}{\omega_0}\right|;$$

λ_0 and γ_0 are the real and imaginary parts of $E_0 - \varepsilon_p$ in the first approximation (λ_0 and γ_0 are of order $e^2\omega_0$).

For nonrelativistic velocities ($v \ll 1$) the radiative corrections are small,

$$\Delta^t = \frac{2}{3}v^2 \ln\frac{4}{|\xi_0|^2}, \qquad |\xi_0^l|^2 = \frac{e^4}{v^2}\left(\frac{\pi^2}{16} + \ln^2\frac{\chi_{\max}v}{\omega_0}\right), \tag{13}$$

For ultrarelativistic velocities we obtain*

$$\Delta^t = 2\ln^2\frac{\varepsilon_p}{m} + 2\left(\ln\frac{2\varepsilon_p}{m} - 1\right)\left(\ln\frac{m^2}{\varepsilon_p^2\xi_0^2} - 1\right) + 0.1696 \quad \text{for } \frac{\varepsilon_p}{m} \ll \frac{1}{|\xi_0|}. \tag{14}$$

$$\Delta^t = \frac{7}{2} - \frac{\pi^2}{6} + \frac{1}{2}\ln^2\frac{1}{2.72\,\xi_0^2} = 1.87 + 2\ln^2\frac{1}{1.65\,\xi_0} \quad \text{for } \frac{\varepsilon_p}{m} \gg \frac{1}{|\xi_0|}. \tag{15}$$

It should be noted that the radiative corrections become appreciable at $\varepsilon_p/m \gtrsim 10^2$. The formulas obtained describe the macroscopic part of the radiative corrections to ionization losses. The ionization curve–the momentum falls by 5-7%, which, as it turned out, agrees with experiment ([10]).**

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CITED LITERATURE

1. I. E. Tamm, I. M. Frank, *DAN*, **14**, 107 (1937).
2. I. E. Tamm, *J. Phys. USSR*, **1**, 439 (1939).
3. E. Fermi, *Phys. Rev.*, **57**, 485 (1940).
4. V. L. Ginzburg, *ZhETF*, **10**, 589 (1940).
5. V. N. Tsytovich, *ZhETF*, No. 2 (1962).

6. E. S. Fradkin, Dissertation, Lebedev Physical Institute, Academy of Sciences of the USSR, 1960; Sh. M. Kogan, *DAN*, **126**, 546 (1959); V. L. Bonch-Bruevich, *DAN*, **126**, 539 (1959).

7. V. P. Silin, A. A. Rukhadze, *Electromagnetic Properties of Plasma and Plasma-like Media*, Moscow, 1961.

8. N. Tsytovich, *ZhETF*, **40**, 1776 (1961).

9. V. N. Tsytovich, *ZhETF*, No. 3 (1962).

10. G. B. Zhdanov, M. Novak, M. I. Tretyakova, Proceedings of the Conference on Photoemulsions, Moscow, 1960.

* For ultrarelativistic velocities, λ_0^l and λ_0^t almost exactly compensate one another, so that their difference does not depend on the natural frequencies of the medium ω_s (an effect analogous to the density effect for losses) and decreases with increasing energy as m/ε_p ([5]). Therefore in (14) and (15) $|\xi_0| = \gamma_0/\omega_0$. It should be emphasized that γ_0, in contrast to the energy loss W_0, depends on the natural frequencies of the medium.

** A detailed comparison will be published separately.

Note: Figure translations are in progress. See original paper for figures.