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Abstract

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SOME CRITERIA FOR GENERALIZED SOLVABILITY AND SPECIALNESS OF πd-GROUPS

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§ 1. The present work is a continuation of our previous investigations of the influence of the number of classes of noninvariant conjugate πd-subgroups of a group G, for a given number of distinct prime π-divisors of the order, on its properties, published in article ([1]).

The notation and terminology introduced by us in ([1]) are also used in the present note: G is a finite group of order $g = mn$, where $m > 1$ is the largest π-Sylow divisor ([2]) of the order g, $n \geqslant 1$, and, for $n > 1$,

$$n = q_1^{\beta_1} q_2^{\beta_2} \dots q_s^{\beta_s}$$

is the canonical decomposition; a πd-group is a group whose order is divisible by some $p \in \pi$ ([3]); t is the number of distinct prime π-divisors of the order; r is the number of classes of noninvariant πd-subgroups of the group G.

Definition 1. Put $r - t = \lambda$. Then the numbers r and $\lambda + 2$ will be called respectively the π-rank and the π-type of the group.

Theorem A (S. A. Chunikhin). *If the group G is π-separable and if m is such a divisor of its order g that $m > 1$ and all prime divisors of m belong to π, and, moreover, $(g/m, m) = 1$, then G has at least one solvable subgroup of order m, and all subgroups of order m are conjugate to it* ([4]).

Definition 2. Let m be the largest π-Sylow divisor of the order $g = mn$ of the group G. If G contains a subgroup N of order n and a solvable subgroup M of order m, and all subgroups of order n from G are conjugate to N, while all subgroups of order m from G are conjugate to M, then G will be called a group of type $\pi - 2$.

Theorem B (S. A. Chunikhin). *Every π-solvable group is a group of type $\pi - 2$* ([5]).

Trofimov-Toropov Lemma. *If G is a nonspecial πd-group, then for it $\lambda \geqslant -1$* ([6,7]).

§ 2. In the present work, with the aid of the theorems of O. Yu. Schmidt ([8]), Burnside ([9]), and S. A. Chunikhin ([4]), as well as the Trofimov-Toropov lemma ([6,7]) and P. I. Trofimov' s "method of intersections" ([10], Lemmas 1 and 1′), the following main results have been obtained.

Theorem 1. *Every πd-group G of π-type 4, for which $m \neq p_1^{\alpha_1}, p_1 p_2$ when $n > 1$, where p_1, p_2 are distinct prime numbers from π, is π-solvable.*

Theorem 2. *Every πd-group G of π-type 4, for which $m = p_1^{\alpha_1}$, $n > 1$, is π-separable. If, however, $m = p_1 p_2$, $n > 1$, and G is not π-separable, then it will be a simple group.*

From Theorems 1-3 of article ([1]) and the theorems formulated here there follow, as corollaries:

Theorem 3. *If a πd-group G is not π-separable, then its π-type is not less than 4.*

Theorem 4. *Every πd-group G whose π-type is not greater than 3 is π-solvable, with the exception of the case $m = p$ for π-type 3. In this exceptional case G is π-separable.*

Theorem 5. Every πd-group G of π-rank not greater than 4 is π-solvable, with the exception of the cases: $m = p$, $n > 1$, when it is of π-type 3, and $m = p_1^{\alpha_1}$, $p_1 p_2$ for $n > 1$, when it is of π-type 4.

Hence, as particular cases, the main results of E. N. Toropov (see ([7]), Theorems 2–6) and the results of O. Yu. Schmidt and P. I. Trofimov ([8,11,6]), concerning the solvability of groups with a number of classes of non-invariant subgroups not greater than 4, are obtained directly.

Theorem 6. Every πd-group G of π-rank 5 for which $m \neq p_1^{\alpha_1} p_2^{\alpha_2}$ is π-separable.

Relying on Theorem 6 of ([1]), Theorem 1 of the present paper, and Theorem 2 of S. A. Chunikhin ([5]), we finally obtain the following theorem:

Theorem 7. If G is a nonspecial πd-group of π-rank greater than 5, then its π-type is not less than 5.

Hence the following corollary follows:

Corollary. πd-groups of π-types 1, 2, 3, and 4 with π-rank greater than 5 are special.

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Note: Figure translations are in progress. See original paper for figures.