

ON THE MANIFESTATION OF ROCK PRESSURE IN HORIZONTAL WORKINGS

1962

SovietRxiv

View the original and related papers at <https://sovietrxiv.org/items/ru-196201.82035>

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

Abstract

Full Text

CONTINUUM MECHANICS

L. V. ERSHOV

ON THE MANIFESTATION OF ROCK PRESSURE IN HORIZONTAL WORKINGS

(Presented by Academician A. Yu. Ishlinskii, 23 III 1962)

In the study of the problem of rock pressure^(1,2), two directions have emerged: the first proceeds by using special hypotheses, while the second is based on methods of the theory of elasticity and plasticity. At the same time, all investigations connected with the second direction are based on consideration of problems of either elastic or elastic-plastic equilibrium of bodies.

In the present work an attempt is made to bring the apparatus of the stability theory of elastic-plastic bodies^(3,4) to bear on the study of the manifestation of rock pressure in horizontal workings.

Assuming that the working is located at a sufficiently great depth h below the day surface, we replace the rock mass, as is usually done, by a weightless infinite plane with an opening of radius a , along whose contour a pressure p is applied. At infinity the stresses in the plate tend to the value γh (γ is the unit weight of the rock), i.e., the initial stress state in the mass (before the working is driven) is taken to be hydrostatic.

Under the action of the applied loads an elastic-plastic state arises in the mass. For some value of the pressure $p = p^*$ (which we shall call critical), along with the axisymmetric one, other forms of equilibrium of the working may also exist. In other words, the manifestation of rock pressure is regarded as a process of loss of stability of the mass, leading to a change in the shape of the working. In order to avoid this, it is necessary to install a supporting lining in the working, from whose side a pressure of not less than p^* must act on the rock.

Assuming that σ_n and τ_n are, respectively, the normal and tangential components of stress, we write the plasticity condition in the form⁽⁵⁾

$$\max\{|\tau_n| - (\sigma_n + H) \operatorname{tg} \rho\} = 0, \quad (1)$$

where $H = k \operatorname{ctg} \rho$; k is the cohesion coefficient; ρ is the angle of internal friction of the rock.

Let us denote by r, θ dimensionless polar coordinates (all quantities having the dimension of displacement are referred to the working radius a); by $\sigma_r, \sigma_\theta, \tau_{r\theta}$ the stress components; by $\varepsilon_r, \varepsilon_\theta, \varepsilon_{r\theta}$ the strain components; by G the shear modulus,

by ν Poisson's ratio; by u_r, u_θ the displacements, respectively, along the radius r and in the direction perpendicular to it; and by β_0 the dimensionless radius of the plastic zone. We shall agree to indicate, by square brackets attached to indices, quantities referring to the plastic region, and by parentheses those referring to the elastic region (where indication of the region is not required, the brackets are omitted).

To solve the problem it is necessary to use the equations of equilibrium

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r} \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0; \quad \frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} + \frac{2\tau_{r\theta}}{r} = 0, \quad (2)$$

the plasticity condition

$$f(\sigma_r, \sigma_\theta, \tau_{r\theta}) = k \quad (k = \text{const})$$

(in our case it has the form (1)); the relations of the law of plastic flow

$$\varepsilon_{ij} = \lambda \frac{\partial f}{\partial \sigma_{ij}}; \quad (3)$$

the boundary conditions and the conditions of conjugacy of the solutions in the elastic and plastic regions.

Using (1)–(3), the boundary conditions, the conjugacy conditions, and also the general equations of the theory of elasticity, it is not difficult to obtain that the elastic-plastic state of the mass in the axisymmetric case is determined by the relations

$$\sigma_{[r]}^0 = -H + (H + p)r^{\alpha_2}; \quad \sigma_{[\theta]}^0 = -H + \alpha_1(H + p)r^{\alpha_2}; \quad \tau_{[r\theta]}^0 = 0; \quad (4)$$

$$u_{[r]}^0 = \frac{\mu \beta_0^{1+\alpha_1}}{r^{\alpha_1}}; \quad u_{[\theta]}^0 = 0; \quad \beta_0^{\alpha_2} = \frac{2(\gamma h + H)}{(1 + \alpha_1)(H + p)};$$

$$\sigma_{(r)}^0 = \gamma h - \frac{\alpha_2(H + p)\beta_0^{\alpha_2+2}}{2} \frac{1}{r^2}; \quad \sigma_{(\theta)}^0 = \gamma h + \frac{\alpha_2(H + p)\beta_0^{\alpha_2+2}}{2} \frac{1}{r^2}; \quad (5)$$

$$\tau_{(r\theta)}^0 = 0; \quad u_{(r)}^0 = \frac{1}{2G} r \left[\frac{\alpha_2(H + p)\beta_0^{\alpha_2+2}}{2} \frac{1}{r^2} + \gamma h(1 - 2\nu) \right]; \quad u_{(\theta)}^0 = 0.$$

Here

$$\alpha_1 = \frac{1 + \sin \rho}{1 - \sin \rho}, \quad \alpha_2 = \frac{2 \sin \rho}{1 - \sin \rho}; \quad \mu = \frac{1}{2G} [\sin \rho(\gamma h + H) + \gamma h(1 - 2\nu)].$$

In the general case, when the rock mass loses stability, the contour equation may be represented as $r = 1 + f(\theta)$. Let us take the case $f(\theta) = d \cos \theta$ ($d = \text{const}$).

We shall seek the solution of the problem in the form

$$\sigma_r = \sigma_r^0 + \sigma'_r; \quad \sigma_\theta = \sigma_\theta^0 + \sigma'_\theta; \quad \tau_{r\theta} = \tau_{r\theta}^0 + \tau'_{r\theta}; \quad u_r = u_r^0 + u'_r; \quad u_\theta = u_\theta^0 + u'_\theta.$$

The components of the perturbed state satisfy the boundary conditions

$$\sigma'_r + \frac{d\sigma_r^0}{dr} u' = 0; \quad \tau'_{r\theta} - (\sigma_\theta^0 - \sigma_r^0) \frac{\partial u'}{\partial \theta} = 0 \quad \text{for } r = 1, \quad (6)$$

and at infinity all the primed components tend to zero.

The conditions of conjugacy of stresses and displacements are satisfied on the boundary of the plastic zone

$$\sigma'_{[r]} = \sigma'_{(r)}; \quad \tau'_{[r\theta]} = \tau'_{(r\theta)}; \quad u'_{[r]} = u'_{(r)}; \quad u'_{[\theta]} = u'_{(\theta)} \quad \text{for } r = \beta_0. \quad (7)$$

Linearizing the plasticity condition (1), using the equilibrium equation (2) and the linearized relations (3) to determine the stress and displacement components of the perturbed state in the plastic region, we obtain the equations

$$r^2 \frac{\partial^2 \tau'_{r\theta}}{\partial r^2} + (3 - \alpha_2)r \frac{\partial \tau'_{r\theta}}{\partial r} - \alpha_1 \frac{\partial^2 \tau'_{r\theta}}{\partial \theta^2} - 2\alpha_2 \tau'_{r\theta} = 0; \quad (8)$$

$$\frac{\partial u'_\theta}{\partial r} - \frac{u'_\theta}{r} + \frac{1}{r} \frac{\partial u'_r}{\partial \theta} = \frac{(1 + \alpha_1)\mu\beta_0^{1+\alpha_1}}{\alpha_2(H+p)r^{2\alpha_1}} \tau'_{r\theta}; \quad (9)$$

$$\frac{\partial u'_r}{\partial r} + \alpha_1 \left[\frac{1}{r} \frac{\partial u'_\theta}{\partial \theta} + \frac{u'_r}{r} \right] = 0. \quad (10)$$

We seek the solution of equation (8) in the form

$$\tau'_{r\theta} = \Phi(r) \sin \theta.$$

Putting $u'_r = \frac{\alpha_1}{r^{\alpha_1}} \frac{\partial \psi}{\partial \theta}$, $u'_\theta = -\frac{1}{r^{\alpha_1-1}} \frac{\partial \psi}{\partial r}$, thereby satisfying equation (10), from (9) we obtain an equation for determining the function $\psi(r, \theta)$.

Fig. 1

Figure 1: Fig. 1

We give the expressions for the components of stresses and displacements of the perturbed state in the plastic region:

$$\sigma'_{(r)} = \frac{1}{\alpha_1 r} [C_1 + \alpha_1 C_2 r^{\alpha_2}] \cos \theta; \quad \sigma'_{(\theta)} = \frac{1}{r} [C_1 + \alpha_1 C_2 r^{\alpha_2}] \cos \theta; \quad (11)$$

$$\tau'_{(r\theta)} = \frac{1}{r} [C_1 + C_2 r^{\alpha_2}] \sin \theta;$$

$$u'_{(r)} = \left[\frac{\alpha_1 D_1}{r^{\alpha_1-1}} + \alpha_1 D_2 - \frac{AC_1}{r^{2\alpha_1}} - \frac{\alpha_1 AC_2}{r^{\alpha_1+1}} \right] \cos \theta;$$

$$u'_{(\theta)} = - \left[\frac{D_1}{r^{\alpha_1-1}} + \alpha_1 D_2 + \frac{AC_1}{r^{2\alpha_1}} + \frac{AC_2}{r^{\alpha_1+1}} \right] \sin \theta. \quad (12)$$

Here $A = \mu\beta_0^{1+\alpha_1}/2\alpha_2(H+p)$; C_1, C_2, D_1, D_2 are arbitrary constants.

The components of stresses and displacements of the perturbed state in the elastic region are determined according to (6). They have the form:

$$\sigma'_{(r)} = -\frac{2B}{r^3} \cos \theta; \quad \sigma'_{(\theta)} = 2\frac{B}{r^3} \cos \theta; \quad \tau'_{(r\theta)} = -\frac{2B}{r^3} \sin \theta; \quad (13)$$

$$u'_{(r)} = \frac{B}{2G} \frac{1}{r^2} \cos \theta, \quad u'_{(\theta)} = \frac{B}{2G} \frac{1}{r^2} \sin \theta, \quad (14)$$

where B is an arbitrary constant.

Using the condition of equilibrium of the loads acting on the boundaries of the elastic zone, we find that $C_1 = 0$.

Fig. 1

Substituting (11)–(14) into relations (6)–(7), and using (4), to determine the constants C_2, D_1, D_2, B we obtain a homogeneous linear system of 4 equations, which has a nontrivial solution. Consequently, the determinant of this system will be equal to zero. From the latter condition we obtain an equation for determining the critical parameter β_0 :

$$\alpha_1 \alpha_2 \mu \beta_0^2 + \alpha_1 \left[\frac{2\alpha_2 q}{1 + \alpha_1} - (1 + \alpha_1) \mu \right] = [2(\alpha_2 - \mu \alpha_1) + \alpha_2 q] \frac{1}{\beta_0^{\alpha_2}} \quad (15)$$

$$\left(q = \frac{\gamma h + H}{G} \right).$$

Equation (15) has a unique real positive root $\beta_0 \geq 1$. From the value of β_0 found from (15), according to (4), we find the rock pressure on the support

$$p^* = \frac{2(\gamma h + H) - \beta_0^{\alpha_2}(1 + \alpha_1)H}{(1 + \alpha_1)\beta_0^{\alpha_2}}. \quad (16)$$

For the case of plastic clays (according to (7) $\rho = 15^\circ$, $k = 5 \text{ t/m}^2$, $G = 0.1 \cdot 10^5 \text{ t/m}^2$, $\gamma = 2.5 \text{ t/m}^2$, $\nu = 0.25$), Fig. 1 gives graphs of the variation of β_0 and p^* as functions of the depth h at which the excavation is driven.

The author expresses his deep gratitude to Acad. A. Yu. Ishlinsky for his attention and a number of valuable comments.

Moscow Mining
Institute

Received
5 VI 1961

REFERENCES

1. Collection *Studies of Rock Pressure*, 1960.
2. Collection of translations *Problems in the Theory of Rock Pressure*, 1961.
3. L. S. Leibenzon, *Collected Works*, 1, Publishing House of the Academy of Sciences of the USSR, 1951.
4. A. Yu. Ishlinsky, *Ukrainian Mathematical Journal*, 6, No. 2 (1954).
5. V. V. Sokolovsky, *Statics of a Granular Medium*, 1960.
6. K. B. Bitsenko, R. Grammel, *Technical Dynamics*, 1, 1950.
7. K. V. Ruppeneyt, *Mechanical Properties of Rocks*, 1956.

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.