Cybernetics and Control Theory

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Abstract

Full Text

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REDUCING THE NUMBER OF STATES OF ONE CLASS OF FINITE AUTOMATA

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1. In considering a finite automaton we shall use the following notation (1): $\varkappa(p)$ is the internal state of the automaton, $\rho(p)$ is the input state, $\lambda(p)$ is the output state, where p denotes time intervals corresponding to the duration of cycles determined by the states of the automaton.

The state of the automaton, which we shall denote by $\mu(p)$, is defined as follows:

$$\mu(p) = \Phi(\rho(p); \varkappa(p)). \tag{1}$$

In contrast to the automata described in (1), one may consider automata whose behavior is determined by the following two equations:

$$\varkappa(p+1) = \psi\{d[\mu(p-1)]\} = \psi\{d[\rho(p-1); \varkappa(p-1)]\}; \tag{2}$$

$$\lambda(p) = \Gamma[\rho(p); \varkappa(p)], \tag{3}$$

where d, the transition operator (2), is used here to denote a change in the state of the automaton, and $d[\mu(p-1)]$ denotes the change in the state of the automaton in passing from cycle $p-1$ to cycle p, which may be caused either by a change in the input state or by a change in the internal state of the automaton*.

The operating conditions of a finite automaton may be written in the form of a transition matrix (3) (the transition matrix in (3) is used to describe the operation of a synchronous automaton). In the present paper we consider: a) the use of the transition matrix to describe the operation of an asynchronous automaton for which the duration of the cycle T, determined by the input state, is a variable quantity, and it is assumed that the transition time from one internal state of the automaton to another, i.e., the duration of the cycle p, is less than T; b) a method for reducing the number of internal states of an automaton whose behavior is described by equations (2), (3).

2. A characteristic feature of the transition matrix describing the operation of an asynchronous finite automaton is, obviously, that its main diagonal

must always be filled, and the elements of the main diagonal correspond to stable states of the automaton (4); the remaining elements of the matrix correspond to unstable states of the automaton (4) and indicate those input states that transfer the automaton from one internal state to another.

Having described the operating conditions of an asynchronous automaton by a transition matrix, one may use the known method of its compression (3, 5), which corresponds to a reduction in the number of internal states of the automaton.

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* In this note we consider automata in which a change of state is associated only with a change of the input state, i.e., automata described by the equation $\varkappa(p+1) = \psi[\varkappa(p-1), d(\rho(p-1))]$ and by equation (3), and in this case $\varkappa(p-1) = \varkappa(p)$.

Let, for example, the operating conditions of the finite automaton be described by the matrix M_1

$$M_1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{array} \left[\begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ A,0 & B,0 & & & & & & & & & & \\ & B,0 & A,0 & & & & & & & & & \\ & & A,0 & B,0 & & & & & & & & \\ & & & B,0 & C,\alpha & & & & & & & \\ & & & & C,\alpha & B,0 & & & & & & \\ & & & & & B,0 & A,0 & & & & & \\ & & & & & & A,0 & B,\beta & & & & \\ & & & & & & & B,\beta & A,0 & & & \\ & & & & & & & & A,0 & B,0 & & \\ & & & & & & & & & B,0 & C,\beta & \\ & & & & & & & & & & C,\beta & B,0 \\ & & & & & & & & & & & B,0 \end{array} \right].$$

After compressing M_1 by method (5), we obtain M_2, describing an automaton equivalent to the original one, with the minimal number of internal states in the class of automata represented by the equation

$$\varkappa(p) = \chi[\rho(p); \varkappa(p-1)] \tag{4}$$

and by equation (3):

$$M_2 = \begin{array}{c} 1' \\ 2' \\ 3' \\ 4' \\ 5' \end{array} \left[\begin{array}{ccccc} 1' & 2' & 3' & 4' & 5' \\ A,0 \vee B,0 & C,\alpha & & & \\ & C,\alpha \vee B,0 & A,0 & & \\ & & A,0 & B,\beta & \\ & & & B,\beta & A,0 \\ & & & & A,0 \vee B,0 \vee C,\beta \end{array} \right].$$

3. In the case of using an automaton described by equations (2), (3), one can carry out a further compression of the transition matrix and obtain the matrix M_3:

$$M_3 = \begin{array}{c} 1'' \\ 2'' \\ 3'' \\ 4'' \end{array} \left[\begin{array}{cccc} 1'' & 2'' & 3'' & 4'' \\ A,0 \vee B,0 & B,0/C,\alpha & & \\ & C,\alpha \vee B,0 & B,0/A,0 & \\ & & A,0 \vee B,\beta & B,\beta/A,0 \\ & & & A,0 \vee B,0 \vee C,\beta \end{array} \right].$$

In M_3, the elements of the main diagonal still correspond to stable states of the automaton, while the remaining elements of the matrix indicate such changes of the automaton' s states as cause a transition to a new internal state. For example, the element $1'', 2''(B,0/C,\alpha)$, formed from such terms as the element $1'', 1''(B,0)$ and the element $2'', 2''(C,\alpha)$, which make it possible to preserve the input-state sequence specified by the original matrix M_1, indicates that when the input state changes from B to C, the automaton will pass into the new internal state $2''$.

The transition from M_2 to M_3 is based on combining certain stable states of the automaton and eliminating those internal states of the automaton in which the element of the main diagonal has turned out to be unfilled, which indicates that the automaton will never be in this state.

The merging of states is possible, obviously, only in those cases where, in each row of the newly obtained matrix, the same input states correspond to one and the same output states, and all sequences of input states that existed in the original automaton are carried out. In the example under consideration (M_2), the internal state $4'$ can be eliminated by merging the states corresponding to the matrix elements $3', 3'$ and $4', 4'$. From the newly obtained internal state $3''$, the automaton must now pass directly to state $5'$ when the input state changes from B to A, to

which indicates the corresponding element of the matrix $M_4(B,\beta/A,0)$:

$$M_4 = \begin{array}{c} 1' \\ 2' \\ 3'' \\ 5'' \end{array} \begin{bmatrix} A,0 \vee B,0 & C,\alpha & & \\ & C,\alpha \vee B,0 & A,0 & \\ & & A,0 \vee B,\beta & B,\beta/A,0 \\ & & & A,0 \vee B,0 \vee C,\beta \end{bmatrix} \begin{array}{cccc} 1' & 2' & 3'' & 5'' \end{array}$$

Since in one and the same row of the matrix there must not be contradictory values of output states for the same input states, in our case one cannot exclude, for example, state $3'$, which would lead to M_5.

$$M_5 = \begin{array}{c} 1' \\ 2'' \\ 4' \\ 5' \end{array} \begin{bmatrix} A,0 \vee B,0 & C,\alpha & & \\ & C,\alpha \vee B,0 \vee A,0 & A,0/B,\beta & \\ & & B,\beta & A,0 \\ & & & A,0 \vee B,0 \vee C,\beta \end{bmatrix} \begin{array}{cccc} 1' & 2'' & 4' & 5' \end{array}$$

Further reduction of the number of internal states in the matrix M_4 is impossible; therefore, by renumbering the states, we finally obtain the matrix M_3.

In the case where an element of the main diagonal is a disjunction of several values of input states, the corresponding states can be combined separately with different states. For example, from M_6:

$$M_6 = \begin{array}{c} 1' \\ 2' \\ 3' \\ 4' \\ 5' \\ 6' \end{array} \begin{bmatrix} A,0 & B,\alpha & & C,\gamma & & \\ & B,\alpha \vee C,\beta & A,0 & & & \\ & & A,0 & B,\beta & & C,\alpha \\ & & & C,\gamma \vee B,\beta & A,0 & \\ & C,\beta & & & A,0 & B,\gamma \\ A,0 & & & & & B,\gamma \vee C,\alpha \end{bmatrix} \begin{array}{cccccc} 1' & 2' & 3' & 4' & 5' & 6' \end{array}$$

we obtain M_7:

$$M_7 = \begin{array}{c} 1'' \\ 2'' \\ 3'' \end{array} \begin{bmatrix} A,0 \vee B,\alpha \vee C,\gamma & B,\alpha/A,0 & C,\gamma/A,0 \\ C,\alpha/A,0 & A,0 \vee B,\beta \vee C,\alpha & B,\beta/A,0 \\ B,\gamma/A,0 & C,\beta/A,0 & A,0 \vee C,\beta \vee B,\gamma \end{bmatrix} \begin{array}{ccc} 1'' & 2'' & 3'' \end{array}$$

In conclusion it should be noted that the process of synthesizing a finite automaton described by equations (2), (3) includes not only minimization of the number of internal states, but also determination of the Boolean functions Γ ([3]) and of the potential-pulse forms ([2,4]) needed in solving equation (2), as well as their integration ([4,6]).

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Note: Figure translations are in progress. See original paper for figures.