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Abstract

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GEOPHYSICS

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THE SHAPE OF THE SURFACE OF A LIQUID LOSING WEIGHT

Modern investigations of the vicinity of the Earth are posing new physical problems, which previously might have seemed abstract. In particular, it has become necessary to analyze the behavior of matter in a state of weightlessness and on the paths leading to this state. The lifelong memory of Plateau' s well-known experiments is always associated with the idea of how oil, suspended in a mixture of water and alcohol, assumes a spherical shape, for which the free surface of the given volume of liquid is minimal. At a press conference on August 21, 1962, at Moscow University, pilot-cosmonaut P. R. Popovich reported the first observations in the world of how water behaves in a flask that is actually in a state of weightlessness. The water, which at first occupied the lower hemisphere, assumed the form of a hollow spherical shell, inside which air was located. It is easy to see that in this case the interface between the water and the air increased by a factor of 2.52. It is equally obvious that the cause of such behavior of water is intermolecular forces, which cause ideal wetting of the glass surface by water.

Let us trace how the surface of water in a glass vessel should change as weight is gradually reduced to zero. For simplicity of analysis, let us suppose that the water is in a cylindrical glass of radius R.

Let us place the origin of a cylindrical coordinate system at the point of intersection of the axis of the glass with the water–air interface. In the combined field of gravity and the centrifugal force that arises on approach to the stationary orbit of a spacecraft, the acceleration is characterized by some value $g_1 < g$. Hence the general equation of equilibrium of the water surface with principal radii of curvature r_1, r_2, in the presence of surface tension σ, must be written in the form:

$$\delta g_1 z = \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right), \tag{1}$$

where δ is the density of the liquid, and z is the height of the column of liquid above the coordinate plane. Let us also write the expression for the capillary constant

$$h^2 = \frac{2\sigma}{\delta g_1} \tag{2}$$

and regard h as a certain universal scale parameter, making it possible to obtain relations between dimensionless quantities. Put

$$\frac{z}{h} = \zeta; \qquad \frac{r_1}{h} = \rho_1; \qquad \frac{r_2}{h} = \rho_2. \tag{3}$$

Then instead of (1) one may write:

$$\zeta = \frac{1}{2}\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right). \tag{4}$$

This equation is valid both under terrestrial conditions (when $g_1 = g$) and at very small, but finite, weight, characterized by various values of g_1. From (4) arise the differential equations of the surface

of rotation, bounding the water and the air. Let x denote the distance from some point of this surface to the coordinate axis, which serves as the axis of rotation of the contour, and let φ denote the angle between this axis and the normal at the same point of the surface. Passing from x to the corresponding dimensionless coordinate $x/h = \xi$, we express ρ_2 in terms of ξ and φ:

$$\rho_2 = \frac{\xi}{\sin\varphi}. \tag{5}$$

Then, after simple transformations proposed by A. N. Krylov ([1]), we obtain from (4) the system of equations:

$$\frac{d\zeta}{d\varphi} = \rho_1 \sin\varphi; \qquad \frac{d\xi}{d\varphi} = \rho_1 \cos\varphi; \qquad \frac{1}{\rho_1} = 2\zeta - \frac{\sin\varphi}{\xi}. \tag{6}$$

In integrating this system, the decisive boundary conditions will be: (a) the condition of ideal wetting of glass by water, which makes the contact angle equal to zero; (b) the value of the ratio R/h. In calculating the scale parameter from (2) we shall always take $\sigma = 73$ dyn$\cdot$cm^{-1} and $\delta = 1$ g$\cdot$cm^{-3}. In the initial conditions, on the surface of the Earth at rest, we take $g_1 = 981$ cm$\cdot$sec^{-2}. Then, expressing h, as is customary, in millimeters, we obtain on Earth $h = 3.9$ mm. If the diameter of the glass is tens of millimeters, then the ratio R/h is very large compared with unity. The meniscus must have a very small radius of curvature ρ_1 near the wall of the glass and practically pass into a plane at considerable distances ξ from the coordinate axis. Therefore one may neglect $1/\rho_2$ in comparison with $1/\rho_1$ in (4), after which it becomes

$$\zeta = 1/2\rho_1. \tag{7}$$

It is easy to see that this is the last of equations (6), without the second term on the right-hand side. Together with the first equation of system (6), it gives the simple equation

$$\frac{d\zeta}{d\varphi} = \frac{\sin\varphi}{2\zeta}, \tag{8}$$

whose integral is

$$\zeta = 1 - \cos\varphi. \tag{9}$$

This equation is satisfied by the initial profile *a* in Fig. 1, drawn according to the well-known formula for plane problems (see, for example, ([2])). For clarity, the scale of the drawing has been doubled relative to the actual size. On the left the wall of the glass is drawn, and on the right the coordinate axis serving as the axis of rotation of the contours is shown by a dash-dotted line. Along the wall are marked the heights of rise and fall of the water level while the volume remains constant. Thin strokes show the trace of the plane water–air boundary that would arise in the absence of surface tension, which creates the meniscus. The form of curve *a* does not depend on R so long as $R/h \gg 1$.

In flight, at small g_1, R/h rapidly decreases, and integration of system (6) by quadratures becomes impossible. Various methods were used for approximate integration of equations of type (6). For example, A. N. Krylov ([1]) used K. Störmer' s method. However, for our purposes there is no need to carry out the integration anew, since system (6) describes whole families of similar surfaces: it is sufficient to find in the existing literature images of the profiles of the water surface for different values of R/h, and then to make an elementary recalculation of the coordinates of the points of each curve as applied to the new value of R.

For this we used the exact drawings of W. Thomson ([3]), to which A. N. Krylov refers in ([1]). Here, in figures 26, 24, and 25, the boundary condition is determined by the values R/h, respectively, 3.23, 2.27, and 0.91. The contact angle is everywhere equal to zero.

It is easy to verify that, considering the behavior of water in a glass of diameter 110 mm, i.e., taking $R = 55$ mm, we obtain contours similar to the Thomson ones for new values of the scale parameter $h_b = 17$ mm, $h_c = 24.3$ mm, and $h_d = 60.5$ mm. These parameters correspond to a decrease in weight, respectively, by factors of 19, 39, and 240.

Figure 1 shows the profiles of the water surface that arise under such conditions in a glass of diameter 110 mm. The triangular pointers near the wall of the glass mark the upper edge of the meniscus.

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Fig. 1

It remains to construct the limiting profile of water in the same glass under complete loss of weight. In this case equations (1)–(4), which describe equilibrium in the presence of acceleration g_1, lose their meaning. They are replaced by the condition of constancy of pressure in all parts of the free surface of the water in the absence of external forces:

$$1/\rho_1 + 1/\rho_2 = \text{const.} \tag{10}$$

This equation is satisfied by the only surface that at the same time satisfies the boundary condition at the glass: a spherical surface of radius R, tangent to the cylinder. Figure 1 shows the trace of this surface on the plane of the drawing: the circular arc e with its center at the point marked by a small circle on the ordinate axis. It is not difficult to show that in con—

under weightlessness the upper edge of the meniscus cannot remain at rest above the line of tangency of this spherical surface. Indeed, if such a rise occurred, the condition of constancy of the volume of water would force the radii of curvature to change, and in such a way that forces would arise tending to return the meniscus to the state corresponding to requirement (10).

In P. R. Popovich' s experiments the shape of the vessel was spherical. It allowed the upper edge of the meniscus to move until a sphere filled with air arose inside the water shell. Of course, condition (10) was automatically satisfied here.

Both in a glass beaker and in a glass flask, the upper edge of the meniscus is "pulled" to its limiting position by the forces of surface tension applied to this edge. We shall show that it is precisely the work of these forces that creates the increase in surface energy when the free surface of the water increases in both cases.

In the beaker the initial energy was equal to $\pi\sigma R^2$ ergs, and in the state shown by curve e in Fig. 1 it became equal to $2\pi\sigma R^2$, i.e., it increased by $\pi\sigma R^2$ ergs. The surface-tension force $2\pi\sigma R$, applied to the upper edge of the meniscus, performed work along the path from zero to point e. From geometrical relations it follows that, with the volume of water constant, this distance is equal to $0.653R$. Consequently, the work performed is $1.31\pi\sigma R^2$ ergs. It not only compensates the increase in surface energy, but also creates an excess $0.31\pi\sigma R^2$, which is converted into heat*.

Fig. 2

In a spherical flask, when rising from the plane of the equator, the length of the meniscus edge is variable: it is equal to $2\pi R \cos\beta$, where β denotes the angle between the plane of the equator and the radius drawn to the edge of the meniscus. It is easy to show that under these conditions the total work of the surface-tension forces along the path from the equator to the pole (we neglect the area of the neck) is $2\pi\sigma R^2$ ergs. As already mentioned, the free surface of the water under weightlessness increased by a factor of 2.52. Thus the surface energy acquired an increment of $1.52\pi\sigma R^2$ ergs. Here, too, the increase in surface energy was not only compensated, but an excess was produced that was converted into heat; it amounted to $0.48\pi\sigma R^2$ ergs.

P. R. Popovich' s experiments would have proceeded differently if the glass surface had been covered with a hydrophobic film: then there would have been no wetting, and the water would have been bounded on the outside by a sphere that would have intersected the underlying surface at the corresponding contact angle. Figure 2 shows a diagram of the arrangement of a weightless mass of water on the surface of paraffin; the contact angle here is about 107°.

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CITED LITERATURE

[1] A. N. Krylov, *Arkh. fiz. nauk*, **1**, no. 1, 2, 110 (1918). [2] H. Geiger, K. Scheel, *Handb. der Phys.*, **7**, 1927, S. 397. [3] W. Thomson, *Popular Lectures and Adresses*, **1**, p. 40–41 (1891).

* If there were no friction losses, then upon approaching position e this excess would be equal to kinetic energy. It would force the meniscus to rise above e until the ellipsoid of revolution that had arisen stretched to the value of the eccentricity k determined by the equation $k = \sin 1.31\, k$. After this the meniscus would again descend toward the "zero" plane, etc. Such oscillations die out owing to friction, and equilibrium is established in state e.

Note: Figure translations are in progress. See original paper for figures.