PHYSICS

Academician of the Academy of Sciences of the BSSR B. I. STEPANOV and A. M. SAMSON

1962

SovietRxiv

Abstract

Full Text

PHYSICS

Academician of the Academy of Sciences of the BSSR B. I. STEPANOV and A. M. SAMSON

CALCULATION OF THE GENERATION POWER OF A PLANE-PARALLEL LAYER

The calculation of the generation power of a plane-parallel layer can be carried out only in the approximation of nonlinear optics, taking into account the dependence of the absorption coefficient k on the radiation density u. From the data of work ([1]) it follows that, for the optical method of producing a nonequilibrium stationary state,

$$k(u) = \frac{k_0}{1 + \alpha u}. \tag{1}$$

Here α and k_0 are parameters characterizing the material of the layer; α is always positive, while the values of k_0 may have different signs. For small u, $k(u) = k_0$.

An expression for the radiation density inside a plane-parallel layer was obtained by methods of geometrical optics in ([2]). At different distances from the boundaries of the layer the radiation density is not the same. However, for $|kl| < 1$ the difference is small, and for approximate calculations one may use the mean value u:

$$u = u_0(1 - r)\frac{1 - e^{-kl}}{kl}\frac{1}{1 - re^{-kl}}, \tag{2}$$

where r is the reflection coefficient at the boundary of the layer; u_0 is the density of the radiation incident on the layer from outside. Since $u > 0$, expression (2) has meaning only when $re^{-kl} \leq 1$, which imposes restrictions on the possible values of the absorption coefficient.

For given u_0, r, l, α, and k_0, equations (1) and (2) are compatible only for a quite definite value of u (and k). Their solution is not difficult to find graphically.

For the generation regime, i.e., for $u_0 \to 0$, it is not difficult to find an analytic solution of (1) and (2). In this case the plane-parallel layer must possess a considerable reserve of radiant energy. If $u_0 \to 0$, then a finite positive value of (2) can exist only under the condition

$$re^{-kl} \to 1. \tag{3}$$

From (3), (2), and (1) it follows that in the generation regime

$$k = \frac{\ln r}{l}; \tag{4}$$

$$u = \frac{k_0 - k}{\alpha k} = \frac{k_0 l - \ln r}{\alpha \ln r}. \tag{5}$$

The value of the absorption coefficient is negative and is completely determined by the properties of the layer as a resonator. The radiation density inside the layer depends, in addition, on the properties of the material of the layer. Since $u > 0$, generation is possible only when $k_0 l - \ln r < 0$, which is equivalent to the condition

$$re^{-k_0 l} \geq 1. \tag{6}$$

Condition (6) was obtained in work ([3]) within the framework of linear optics; it is also valid in nonlinear optics, determining, for given r and l, the minimum value of k_0 necessary for the onset of generation. Condition (6) is consistent with condition (3), which also ensures the possibility of generation.

It follows from (4) and (5) that, as $r \to 1$, $k \to 0$, $u \to \infty$. The larger the reflection coefficient r, the greater the reserve of energy inside the layer.

The absorption power is related to the absorption coefficient by the usual formula

$$W_{\text{abs}} = cku = \frac{c}{al}(k_0 l - \ln r), \tag{7}$$

where c is the speed of light. In the present case (when condition (6) is satisfied) the absorption power is negative, i.e., an energy-release process is occurring inside the layer. Since in the stationary regime the value of u is conserved, the released energy leaves the layer. The flux through one of the boundaries from an area of 1 cm^2 is

$$S = -\frac{W_{\text{abs}}\, l}{2} = \frac{c(-k_0 l + \ln r)}{2a}. \tag{8}$$

The power of the generated flux depends strongly on the nonlinearity parameter a. The smaller the nonlinearity of the absorption coefficient, the larger the value of S.

Formula (8) is not difficult to obtain in another way as well. The fluxes transmitted and reflected by a plane-parallel layer, within geometrical optics, are equal to (4):

$$S_{\text{tr}} = cu_0 T = cu_0 \frac{(1-r)^2 e^{-kl}}{1-r^2 e^{-2kl}}, \tag{9}$$

$$S_{\text{refl}} = cu_0 R = cu_0 \left[r + \frac{(1-r)^2 r^{-2kl}}{1-r^2 e^{-2kl}} \right]. \tag{10}$$

It follows from (2)–(5) that, as $u_0 \to 0$ and u remains finite,

$$\frac{u_0}{1-re^{-kl}} = \frac{(k_0 l - \ln r)\, r}{a(1-r)^2}. \tag{11}$$

Substituting the value (11) into (9) and (10) and letting u_0 tend to zero, we arrive at (8).

All the results presented above referred to the generation of radiation in the direction normal to the surface of the plane-parallel layer. In other directions generation will be absent. This conclusion, valid for $u_0 = 0$ within the linear approximation, remains in force when nonlinear effects are also taken into account. Indeed, in any bounded plane-parallel layer the number of summed beams remains finite, which leads to a substantial change in the expressions for u and T or R. Thus, for example, the flux transmitted by the layer is determined by the formula

$$cu_0 T = cu_0 \frac{(1-r)^2 e^{-kl/\cos\theta}}{1-r^2 e^{-2kl/\cos\theta}} \left[1 - \left(r^2 e^{-2kl/\cos\theta} \right)^n \right], \tag{12}$$

where n is the number of summed beams, depending on the dimensions of the layer and on the angle of incidence θ of the primary flux cu_0. It follows from (12) that, as $u_0 \to 0$, the outgoing flux is identically equal to zero for any relations between r and l. Generation can exist only when $n \to \infty$.

Using formulas (8) and (5), it is easy to calculate the quality factor of a plane-parallel layer in the generation regime. In accordance with the generally accepted definition, the quality factor is the ratio of the energy stored in the layer to the energy lost during a period:

$$Q = 2\pi \frac{E_{\text{stored}}}{E_{\text{loss}}} \nu = 2\pi \frac{ul}{2S} \nu = 2\pi \frac{l}{\lambda \ln \frac{1}{r}}. \tag{13}$$

For large r, formula (13) simplifies:

Fig. 1

Figure 1: Fig. 1

$$Q = 2\pi \frac{l}{\lambda(1-r)}. \tag{14}$$

The quality factor of a plane-parallel layer, calculated with allowance for nonlinear effects, depends only on the properties of the empty layer (of the empty resonator). The expression for the quality factor of the layer, calculated without allowance for nonlinear effects ([2]), contained a dependence on the absorption coefficient of the layer.

Formula (14) was obtained earlier (see ([5])) by a less rigorous method.

Fig. 1. Dependence of the intensity of the radiation transmitted by the layer, S_{tr}, on the density of the incident radiation for $k_0 l = -0.5$.
1—$r = 1.00$; 2—$r = 0.95$; 3—$r = 0.9$; 4—$r = 0.8$; 5—$r = 0.7$; 6—$r = 0.6$; 7—$r = 0.5$; 8—$r = 0.2$.

Figure 1 gives, as an example, the results of a graphical solution of the system of equations (1) and (2) for $k_0 l = -0.5$ and various r. On the ordinate axis are plotted the values of the flux S_{tr} transmitted by the layer; on the abscissa axis, the values of the incident flux cu_0. The scale along the abscissa and ordinate axes has been chosen up to the factor α. The values of S_{tr} were calculated in accordance with formula (9). It is seen from the figure that for large r satisfying condition (6), the value of S_{tr} at $u_0 = 0$ is finite. As u_0 increases, S_{tr} gradually increases; moreover, the larger r is, the smaller the change in S_{tr} as a function of u_0. For values of r satisfying the condition $re^{-k_0 l} < 1$ (amplification regime), the character of the dependence of S_{tr} on u_0 changes sharply. In contrast to S_{tr} at high r, the values of S_{tr}, when u_0 is varied from 0 to ∞, change within the same limits, taking the minimum value $S_{\text{tr}} = 0$ at $u_0 = 0$. In this case, as r decreases, the dependence of S_{tr} on u_0 approaches a linear one.

Institute of Physics
Academy of Sciences of the BSSR

Received
4 X 1961

REFERENCES

1. B. I. Stepanov, *Fundamentals of the Spectroscopy of Negative Light Fluxes*, Minsk, 1961.

2. B. I. Stepanov, *Dokl. AN BSSR*, **5**, No. 12 (1961).

3. A. M. Prokhorov, *ZhETF*, **31**, 1658 (1958).

4. G. Stokes, *Math. and Phys. Papers*, **4**, 145 (1904).

5. N. G. Basov, O. N. Krokhin, Yu. N. Popov, *Uspekhi fiz. nauk*, **72**, 161 (1960).

Note: Figure translations are in progress. See original paper for figures.