Reports of the Academy of Sciences of the USSR

Academician of the Academy of Sciences of the Uzbek SSR É. I. ADIROV, E. M. KUZNETSOVA

1962

SovietRxiv

Abstract

Full Text

Reports of the Academy of Sciences of the USSR
1962. Vol. 147, No. 4

PHYSICS

Academician of the Academy of Sciences of the Uzbek SSR É. I. ADIROV, E. M. KUZNETSOVA

THE LIFETIME OF NONEQUILIBRIUM CHARGE CARRIERS IN SEMICONDUCTORS WITH LOCAL LEVELS

1. General consideration. The kinetics of electronic processes in a semiconductor with an arbitrary system of local centers is described by the equations

$$\frac{\partial n}{\partial t} - \frac{1}{q}\operatorname{div}\mathbf{i}_n = \sum\sum \left[{}^cB_{kl} - {}^cA_{kl}n\right] N_{kl} + G_n,$$

$$\frac{\partial p}{\partial t} + \frac{1}{q}\operatorname{div}\mathbf{i}_p = \sum\sum \left[{}^vB_{kl} - {}^vA_{kl}p\right] N_{kl} + G_p,$$

$$\partial N_{kl}/\partial t = \left({}^vB_{k\,l-1} + {}^cA_{k\,l-1}n\right) N_{k\,l-1}$$

$$-\left({}^vB_{kl} + {}^cB_{kl} + {}^vA_{kl}p + {}^cA_{kl}n\right) N_{kl} + \left({}^cB_{k\,l+1} + {}^vA_{k\,l+1}p\right) N_{k\,l+1} + G_{kl},$$

$$\partial\rho/\partial t = -\operatorname{div}\mathbf{i}, \qquad \operatorname{div}\mathbf{E} = 4\pi\rho/\chi, \qquad \rho = q\left[p - n - \sum\sum (l - m_k) N_{kl}\right], \tag{1}$$

$$\mathbf{i} = \mathbf{i}_n + \mathbf{i}_p, \qquad \mathbf{i}_p = q\mu_p \mathbf{E} p - qD_p \operatorname{grad} p, \qquad \mathbf{i}_n = q\mu_n \mathbf{E} n + qD_n \operatorname{grad} n,$$

$$G_{kl} = {}^vG_{k\,l-1} - {}^vG_{kl} - {}^cG_{kl} + {}^cG_{k\,l+1}, \qquad G_p - G_n = \sum\sum \left({}^vG_{kl} - {}^cG_{kl}\right).$$

Here n, p, N_{kl} are, respectively, the concentrations of conduction electrons, free holes, and local centers of type k that are in the l-th charge state; G_n, G_p, and G_{kl} are the rates of nonthermal generation of n, p, and N_{kl}. The processes of nonthermal generation of local centers in various ionization states consist of acts of attachment to them of electrons from the valence band (${}^{v}G_{kl}$) and acts of transition of electrons from local centers to the conduction band (${}^{c}G_{kl}$). The summation refers to the lower indices k and l, with the summation over k being performed over all types of local centers, and the summation over l from 1 to r_k, where r_k is the total number of possible charge states of a center of type k. For $l = m_k$, the local center of type k is electrically neutral. The superscript zero everywhere denotes the thermodynamically equilibrium state. The remaining notation is either generally known or is explained in Fig. 1. In what follows, for definiteness, we consider an n-type semiconductor. Let us note:

$${}^{c}B_{k1} = {}^{v}B_{kr_k} = {}^{v}A_{k1} = {}^{c}A_{kr_k} = {}^{c}G_{k1} = {}^{v}G_{kr_k} = 0;$$

$$\left({}^{v}B_{kl}/{}^{v}A_{k\,l+1}\right) = p_0 N^0_{k\,l+1}/N^0_{kl}, \qquad \left({}^{c}B_{k\,l+1}/{}^{c}A_{kl}\right) = n_0 N^0_{kl}/N^0_{k\,l+1}.$$

Consider small disturbances of the thermodynamically equilibrium state of a semiconductor ($n' \equiv n - n_0 \ll n_0$, etc.), produced by a small harmonic signal in the region of fundamental absorption ($G'_n \equiv G'_p = G' e^{j\omega t}$; $G'_{kl} = 0$). Under the condition of neutrality or quasineutrality ($\rho' \simeq 0$), from (1) we obtain ($s = j\omega$):

$$p' \left[1 + \sum\sum (l\Delta_{kl2}/\Delta_k)\right] = n' \left[1 + \sum\sum (l\Delta_{kl1}/\Delta_k)\right],$$

$$N'_{kl} = \left[\Delta_{kl1} n' - \Delta_{kl2} p'\right]/\Delta_k, \qquad \sum_l N'_{kl} = 0, \tag{2}$$

where

$$\Delta_k = \begin{vmatrix} 1 & 1 & 1 & 1 & \ldots & 1 & 1 & 1 \\ \alpha^k_{21} & 1 & \alpha^k_{23} & 0 & \ldots & 0 & 0 & 0 \\ 0 & \alpha^k_{32} & 1 & \alpha^k_{34} & \ldots & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \ldots & \alpha^k_{r_k-1\,r_k-2} & 1 & \alpha^k_{r_k-1\,r_k} \\ 0 & 0 & 0 & 0 & \ldots & 0 & \alpha^k_{r_k\,r_k-1} & 1 \end{vmatrix}, \tag{3}$$

$$\alpha^k_{b\,b-1} = -\left({}^{vB}_{k\,b-1} + {}^{cA}_{k\,b-1} n_0\right) M_{kb}, \qquad \alpha^k_{b\,b+1} = -\left({}^{cB}_{k\,b+1} + {}^{vA}_{k\,b+1} p_0\right) M_{kb},$$

$$M_{kb} = \left[s + {}^{cB}_{kb} + {}^{vB}_{kb} + {}^{cA}_{kb} n_0 + {}^{vA}_{kb} p_0\right]^{-1},$$

Fig. 1

Figure 1: Fig. 1

and the determinants Δ_{klg} are formed from Δ_k by replacing the elements of the l-th column by T^k_{bg} ($b = 1, 2, \dots, r_k$ is the row index), where, for $b \neq 1$,

$$T^k_{bg} = \begin{cases} \left({}^{cA}_{k\,b-1}N^0_{k\,b-1} - {}^{cA}_{kb}N^0_{kb}\right) M_{kb}, & \text{for } g = 1, \\ \left({}^{vA}_{kb}N^0_{kb} - {}^{vA}_{k\,b+1}N^0_{k\,b+1}\right) M_{kb}, & \text{for } g = 2, \end{cases} \tag{4}$$

and for $b = 1$, $T^k_{1g} = 0$.

Relation (2) makes it possible to find p', n', and N'_{ki} as functions of G' and s, and thereby to determine the corresponding lifetimes in a semiconductor with any system of local centers (recombination and nonrecombination, multiply charged and singly charged).

Fig. 1

2. Photoconductivity under uniform excitation. In this case $i'_p = q\mu_{pE}0p'$, $i'_n = q\mu_{nE}0n'$, and coordinate dependences are absent. p' and n' are determined by the equations

$$p' = G'H(s), \qquad n' = G'F(s), \tag{5}$$

where*

$$H(s) = \Big[1 + \sum\sum (l\Delta_{kl1}/\Delta_k)\Big] \Big\{\Big(s + \sum\sum {}^{vA}_{kl}N^0_{kl}\Big) \Big[1 + \sum\sum (l\Delta_{kl1}/\Delta_k)\Big] + \sum\sum \big({}^{vB}_{kl} - {}^{vA}_{kl}p_0\big) [(\Delta_{kl2} - \tag{6}$$

$$F(s) = \Big[1 + \sum\sum (l\Delta_{kl2}/\Delta_k)\Big] \Big\{\Big(s + \sum\sum {}^{vA}_{kl}N^0_{kl}\Big) \Big[1 + \sum\sum (l\Delta_{kl1}/\Delta_k)\Big] + \sum\sum \big({}^{vB}_{kl} - {}^{vA}_{kl}p_0\big) [(\Delta_{kl2} -$$

The lifetimes of holes and electrons in the stationary state of the semiconductor are determined by the equations $G' = p'/\tau^{\text{st}}_p$ and $G' = n'/\tau^{\text{st}}_n$. Therefore,

$$\tau^{\text{st}}_p = H(0), \qquad \tau^{\text{st}}_n = F(0). \tag{7}$$

According to the formulas

$$|f(\omega)|/f(0) = 1/\sqrt{1 + (\omega\tau)^2}, \qquad \operatorname{tg}\arg f = -\omega\tau, \tag{8}$$

corresponding to a process with a constant relaxation time τ, the effective lifetimes of holes and electrons, determined from the amplitude-frequency and phase-frequency characteristics of the concentrations p' and n', are equal to

$$\begin{aligned} \tau_p^a &= \frac{\sqrt{(\tau_p^{\mathrm{st}})^2 - |H(j\omega)|^2}}{\omega |H(j\omega)|}, \qquad & \tau_p^\phi &= -\frac{1}{\omega}\,\mathrm{tg}\arg H(j\omega), \\ \tau_n^a &= \frac{\sqrt{(\tau_n^{\mathrm{st}})^2 - |F(j\omega)|^2}}{\omega |F(j\omega)|}, \qquad & \tau_n^\phi &= -\frac{1}{\omega}\,\mathrm{tg}\arg F(j\omega), \end{aligned} \tag{9}$$

and in the general case are frequency-dependent.

Considering (5) as the Laplace transform of the transition characteristics

$$p'(t) = H(s)G', \qquad n'(t) = F(s)G' \tag{10}$$

and noting that the poles $s_1, s_2, \ldots$ of the functions $H(s)$ and $F(s)$ coincide, we find the characteristic relaxation times of electronic processes in a semiconductor with an arbitrary system of local levels under pulsed excitation:

$$\tau_1 = -1/s_1, \qquad \tau_2 = -1/s_2, \ldots \tag{11}$$

3. **A semiconductor with a system of singly charged local centers.** In the case under consideration $l = 1, 2$, with $N_{k1} = p_k$, $N_{k2} = n_k$. From (6) we find*

$$\begin{aligned} H(s) = {} & [1 + \textstyle\sum {}^cA_k p_{k0} L_k(s)]\Big\{(s + \sum {}^vA_k n_{k0})(1 + \sum {}^cA_k p_{k0} L_k(s)) + \\ & + \textstyle\sum {}^vA_k L_k(s)(\tilde{p}_k + p_0)[{}^cA_k p_{k0} - {}^vA_k n_{k0}] + \\ & + \textstyle\sum\sum {}^vA_k(\tilde{p}_k + p_0) L_k(s) L_m(s)({}^cA_k {}^vA_m p_{k0} n_{m0} - {}^vA_k {}^cA_m n_{k0} p_{m0})\Big\}^{-1}, \\ F(s) = {} & [1 + \textstyle\sum {}^vA_k n_{k0} L_k(s)]\Big\{(s + \sum {}^vA_k n_{k0})(1 + \sum {}^cA_k p_{k0} L_k(s)) + \\ & + \textstyle\sum {}^vA_k L_k(s)(\tilde{p}_k + p_0)[{}^cA_k p_{k0} - {}^vA_k n_{k0}] + \\ & + \textstyle\sum\sum {}^vA_k(\tilde{p}_k + p_0) L_k(s) L_m(s)({}^cA_k {}^vA_m p_{k0} n_{m0} - {}^vA_k {}^cA_m n_{k0} p_{m0})\Big\}^{-1}. \end{aligned} \tag{12}$$

Here $\tilde{p}_k = p_0 n_{k0}/p_{k0}$, $\tilde{n}_k = n_0 p_{k0}/n_{k0}$, and

$$L_k(s) = [s + {}^cA_k(\tilde{n}_k + n_0) + {}^vA_k(\tilde{p}_k + p_0)]^{-1}. \tag{13}$$

---

* The summations refer to the indices k, l, ν, χ and are carried out over k and ν over all types of local levels, and over l and χ from 1 to r_k.

According to (7), in this case

$$
\begin{aligned}
\tau_p^{\text{st}} &= \frac{1 + \sum {}^c A_k p_{k0}/\chi_k}{\sum {}^v A_k n_{k0}(1 + \sum {}^c A_k p_{k0}/\chi_k) + \sum \lambda_k/\chi_k + \sum\sum \mu_{k\zeta}/\chi_k\chi_\zeta}, \\
\tau_n^{\text{st}} &= \frac{1 + \sum {}^v A_k n_{k0}/\chi_k}{\sum {}^v A_k n_{k0}(1 + \sum {}^c A_k p_{k0}/\chi_k) + \sum \lambda_k/\chi_k + \sum\sum \mu_{k\zeta}/\chi_k\chi_\zeta}, \qquad (14) \\
\chi_k &= ({}^v A_k p_0/p_{k0} + {}^c A_k n_0/n_{k0})N_k, \qquad \lambda_k = {}^v A_k p_0 \frac{N_k}{p_{k0}}({}^c A_k p_{k0} - {}^v A_k n_{k0}), \\
\mu_{k\zeta} &= {}^v A_k p_0 \frac{N_k}{p_{k0}}({}^c A_k {}^v A_\zeta p_{k0} n_{\zeta 0} - {}^v A_k {}^c A_\zeta p_{\zeta 0} n_{k0}).
\end{aligned}
$$

* In what follows, summation everywhere refers to lower indices and is carried out over all types of local centers.

These formulas coincide with Guro' s results (1). For a semiconductor with recombination centers of one type and without deep trapping levels, the well-known Shockley-Read formulas (2) follow from (14).

It is seen from expression (12) that the number of poles of the functions $F(s)$ and $H(s)$, and hence the number of characteristic relaxation times of electronic processes in the semiconductor, is greater by one than the number of types of local centers. Representing the coefficients of the polynomial that appears in the denominator of the functions $F(s)$ and $H(s)$ in terms of its roots, we obtain

$$
\sum_k \left[{}^v A_k \left(n_{k0} + \frac{p_0 N_k}{p_{k0}} \right) + {}^c A_k \left(p_{k0} + \frac{n_0 N_k}{n_{k0}} \right) \right] = \sum_i \frac{1}{\tau_i} \approx \frac{1}{\tau_{\text{min}}^{\text{rel}}},
$$

$$
\begin{gathered}
\left\{ \prod_k x_k + \sum_k {}^c A_{kp_{k0}} \prod_{i \neq k} x_i + \sum_k {}^v A_{kn_{k0}} \left(\sum_k \prod_{i \neq k} x_i + \sum_k \sum_\xi {}^c A_{kp_{k0}} \prod_{i \neq k,\xi} x_i \right) \right. \\
\left. + \sum_k \sum_\xi \left(\lambda_k \prod_{i \neq k,\xi} x_i + \sum_\zeta \mu_{k\zeta} \prod_{i \neq k,\xi,\zeta} x_i \right) \right\} \left\{ \sum_k {}^v A_{kn_{k0}} \left(\prod_k x_k + \sum_k {}^c A_{kp_{k0}} \prod_{i \neq k} x_i \right) \right. \\
\left. + \sum_k \left(\lambda_k \prod_{i \neq k} x_i + \sum_\zeta \mu_{k\zeta} \prod_{i \neq k,\zeta} x_i \right) \right\}^{-1} = \sum_i \tau_i \approx \tau_{\text{max}}^{\text{rel}}.
\end{gathered}
\tag{15}
$$

The approximate equalities (15) hold in those cases when the smallest $\tau_{\text{min}}^{\text{rel}}$ and the largest $\tau_{\text{max}}^{\text{rel}}$ relaxation times differ from the remaining τ_i by an order of magnitude. In the case of recombination levels of one type and the absence of deep trapping levels, (15) gives $\tau_{\text{min}}^{\text{rel}}$ and $\tau_{\text{max}}^{\text{rel}}$ as obtained in (3–5).

Finding the limiting values of expressions (9), we obtain: for $\omega \to 0$

$$\begin{aligned} \tau_p^a &= \sqrt{(\tau_{\max}^{\mathrm{rel}})^2 - A^2}, & \tau_p^\phi &= \tau_{\max}^{\mathrm{rel}} - A,\\ \tau_n^a &= \sqrt{(\tau_{\max}^{\mathrm{rel}})^2 - B^2}, & \tau_n^\phi &= \tau_{\max}^{\mathrm{rel}} - B, \end{aligned} \tag{16}$$

where

$$A = \frac{\sum_k \left(\prod_{i\neq k} x_i + \sum_\zeta {}^cA_{kp_{k0}} \prod_{i\neq k,\zeta} x_i\right)}{\prod_k x_k + \sum_k {}^cA_{kp_{k0}} \prod_{i\neq k} x_i}, \qquad B = \frac{\sum_k \left(\prod_{i\neq k} x_i + \sum_\zeta {}^vA_{kn_{k0}} \prod_{i\neq k,\zeta} x_i\right)}{\prod_k x_k + \sum_k {}^vA_{kn_{k0}} \prod_{i\neq k} x_i},$$

for $\omega \to \infty$

$$\tau_p^a = \tau_p^{\mathrm{st}}, \qquad \tau_n^a = \tau_n^{\mathrm{st}}, \qquad \tau_p^\phi = 1/\sum {}^vA_{kn_{k0}}, \qquad \tau_n^\phi = 1/\sum {}^cA_{kp_{k0}}.$$

Physico-Technical Institute
Academy of Sciences of the Uzbek SSR

Received
20 VIII 1962

REFERENCES

[1] G. M. Guro, Dissertation, Phys. Inst. named after P. A. Lebedev, Academy of Sciences of the USSR, 1958.

[2] W. Shockley, W. Read, Phys. Rev., **87**, 835 (1952); *Collection: Semiconductor and Crystal-Phosphor Electronics*, 1953, p. 121.

[3] W. Schottky, Halbleiterprobleme, **2**, Ref. 5a (1955).

[4] A. Hoffman, Halbleiterprobleme, **2**, Ref. 5 (1955).

[5] E. I. Adirovich, G. M. Guro, DAN, **108**, No. 3 (1956).

[6] E. I. Adirovich, *Some Problems in the Theory of Crystal Luminescence*, 1951, Ch. 4.