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PHYSICS

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1962

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Abstract

Full Text

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DRIFT INSTABILITY OF AN INHOMOGENEOUS PLASMA IN A MAGNETIC FIELD

1. As is known, in an inhomogeneous plasma located in a magnetic field there are always drift currents of electrons and ions. By analogy with the usual beam instability, one may think that such currents should lead to the excitation of waves whose phase velocity across the magnetic field is of the order of the drift velocity. Such waves, which may be called drift waves, do indeed exist in an inhomogeneous plasma. Their study was begun in the work of Yu. A. Tserkovnikov ([1]), where, using as an example a plasma in the field of a direct current, it was shown that in the presence of a temperature gradient an excitation of drift waves propagating across the magnetic field may occur. A hydrodynamic treatment leads to approximately the same stability condition ([2]). In the work of L. I. Rudakov and R. Z. Sagdeev ([3]), an oblique drift wave was considered, transforming into an ion-acoustic wave as the angle between the wave vector and the magnetic-field vector decreases. For nearly transverse propagation such a wave has very small damping, and in the presence of a temperature gradient ([3]) or a longitudinal current it may become growing in time.

In the present work, in contrast to those listed above, the effect of the finite Larmor radius of the ions is taken into account (see ([4])).

2. Let us consider an inhomogeneous plasma located in a uniform magnetic field **H**, directed along the z-axis. Restricting ourselves to the one-dimensional case, we shall assume that the equilibrium distribution functions of the electrons f_e and ions f_i depend only on one spatial coordinate x. If the functions f_j vary little over the length of the mean Larmor radius, then in a coordinate system in which the mean electric field is absent we have:

$$f_j = f_{0j}(v_\perp^2, v_z, x) + \frac{v_y}{\Omega_j}\frac{\partial f_{0j}}{\partial x}, \tag{1}$$

where $v^2 = v_x^2 + v_y^2$, $\Omega_j = e_j H/m_j c$, e_j is the charge, and m_j is the mass of a particle of species j.

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We shall consider here waves of longitudinal type propagating across the density gradient. For such waves the electric-field potential may be chosen in the form $\varphi' = \varphi \exp(-i\omega t + ik_z z + ik_y y)$, and then the solution of the kinetic equation for the perturbation of the distribution function f_j' of species j is written as an integral along trajectories (see, for example, ([4])):

$$f_j' = i\varphi \frac{e_j}{m_j} \int_{-\infty}^{0} \mathbf{k} \frac{\partial f_j}{\partial \mathbf{v}(t)} \exp(-i\omega t + ik_z v_z t + ik_y y_j(t))\, dt, \tag{2}$$

where

$$y_j(t) = \int_0^t v_{yj}(t)\, dt = \frac{v_\perp}{\Omega_j} \cos(\Omega_j t - \psi) - \frac{v_\perp}{\Omega_j} \cos\psi,$$

ψ being the azimuthal angle in velocity space. For perturbations with wavelength much greater than the Debye radius, the dispersion equation can be obtained

simply from the quasineutrality conditions $\sum_j \int e_j f_j'\, d\mathbf{v} = 0$. We shall assume that the distribution of electrons and ions over the transverse velocities is Maxwellian, with temperatures $T_{\perp e}$ and $T_{\perp i}$, respectively. Since function (1) does not depend on time, after differentiation with respect to $\mathbf{v}$ it can be taken out from under the time integral, and then the dispersion equation is transformed to the form

$$\begin{aligned} &\sum_j \Bigg\{ \int \Bigg(\frac{k_z}{m_j} \frac{\partial f_{0j}}{\partial v_z} + \frac{k_y}{m_j \Omega_j} \frac{\partial f_{0j}}{\partial x} - \frac{k_y v_{yj}(t)}{T_{\perp j}} f_{0j} \\ &\qquad - \frac{k_y v_{yj}(t) v_y}{m_j \Omega_j} \frac{\partial}{\partial x} \left(\frac{f_{0j}}{T_{\perp j}} \right) \Bigg) e^{-i\omega t + ik_z v_z t + ik_y y_j(t)}\, d\mathbf{v}\, dt \Bigg\} \\ &= i \sum_j \Bigg\{ \int \Bigg[\frac{F_j}{T_{\perp j}} + \sum_n (\omega - k_{zv} z - n\Omega_j + i\nu)^{-1} \\ &\qquad \times \Bigg[\frac{k_z}{m_j} \beta_{nj} \frac{\partial F_j}{\partial v_z} + \frac{k_y}{m_j \Omega_j} \frac{\partial}{\partial x} (\beta_{nj} F_j) \\ &\qquad - \frac{\omega - k_{zv} z}{T_{\perp j}} \beta_{nj} F_j - \frac{n^2 k_y}{m_j \Omega_j} \frac{\partial}{\partial x} \left(\beta_{nj} \frac{F_j}{b_j} \right) \Bigg] \Bigg] dv_z \Bigg\} = 0, \end{aligned} \tag{3}$$

where F_j denotes the distribution function with respect to the longitudinal velocity, $\beta_{nj} = e^{-b_j} I_n(b_j)$; I_n is the Bessel function of imaginary argument; $b_j = k_y^2 T_{\perp j} / m_j \Omega_j^2$; ν is a small positive quantity introduced for the correct bypassing of the poles.

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3. Let us consider oscillations of an isotropic ($T_\perp = T_\parallel$), isothermal ($T_e = T_i$) plasma with Maxwellian distributions of electrons and ions with respect to the longitudinal velocity v_z, and with constant temperature ($dT/dx = 0$). Assuming that the oscillation frequency is much smaller than the ion cyclotron frequency Ω_i, we may neglect, in the sum over n in (3), all terms except the zeroth one. Restricting ourselves, moreover, to oscillations with transverse wavelength much larger than the mean Larmor radius of the electrons, we set $b_e = 0$. Under these assumptions equation (3) reduces to

$$(z + a)\beta Y(z) - p(z - \alpha)Y(pz) = 2, \tag{4}$$

where

$$z = \frac{\omega}{k_z}\sqrt{\frac{m_i}{2T}}, \quad b = k_y^2\rho_i^2 = \frac{k_y^2 T}{m_i \Omega_i^2}, \quad p = \sqrt{m_e/m_i} \ll 1, \quad \alpha = -\sqrt{\frac{b}{2}}\times$$

$$\times\frac{1}{k_{za}}, \quad \beta = e^{-b}I_0(b); \quad I_0 \text{ is the Bessel function of imaginary argument;} \quad Y(z) =$$

$$= 2e^{-z^2}\int_0^z e^{t^2}\,dt - i\sqrt{\pi}e^{-z^2};$$

a is the characteristic length over which the mean plasma density n_0 changes appreciably: $a^{-1} = d\ln n_0/dx$.

Investigation of equation (4) shows that for $\alpha < 0$, when the phase velocity of the wave is directed toward the ion drift, the wave is purely damped; moreover, its damping is exponentially small for $\alpha p \gg 1$. Such waves may be excited by longitudinal beams of electrons or ions. As for waves propagating toward the electron drift ($\alpha > 0$), for their growth in time it is sufficient merely that a density gradient be present.

Let us first consider the region $b \gg 1$. For sufficiently large α, i.e., small k_z, z will be sufficiently large, and then approximately $Y(z) \simeq 1/z$. If, on the other hand, $p\alpha \ll 1$, then pz will also be considerably smaller than unity.

In this case $Y(pz) \simeq -i\sqrt{\pi}$, and from (4) we obtain:

$$\frac{\mathrm{Im}\,\omega}{k_y v_0} \simeq \frac{\beta\sqrt{2\pi b}\,p}{k_z a}\frac{1-\beta}{(2-\beta)^2} \ll \frac{\mathrm{Re}\,\omega}{k_y v_0} \simeq \frac{\beta}{2-\beta}, \tag{5}$$

where $v_0 = T/am_i\Omega_i$ is the drift velocity.

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Fig. 1 and Fig. 2

Figure 1: Fig. 1 and Fig. 2

As we see, the growth increment of small perturbations $\operatorname{Im}\omega$ increases as $k_z a$ decreases. This increase continues until the quantity pa becomes of order unity and it is no longer possible to use the expansion for $Y(pz)$. The results of a numerical calculation of the increment near the maximum for three values of b ($b = 0.02$, $b = 1$, $b = 10$) are shown in Fig. 1. Along the abscissa is plotted the dimensionless quantity $k_z a/p\sqrt{b}$, and along the ordinate the dimensionless increment $\gamma = \operatorname{Im}\omega\sqrt{b}/k_y v_0$.

Fig. 1 Fig. 2

The increment γ, due to excitation by resonant electrons, i.e., arising through Landau damping with the opposite sign, decreases in proportion to b^2 for small b, as is evident, for example, from (5). However, for very small b ($b \ll 0.1$) an instability of hydrodynamic character arises for perturbations with a very large wavelength along z ($ap \gg 1$). For such perturbations one may use the asymptotic expansion

$$Y(pz) \simeq \frac{1}{pz} + \frac{1}{2p^3z^3},$$

and then from (4) we obtain:

$$\frac{z-\alpha}{z+\alpha} = 2p^2bz^2. \tag{6}$$

It is not difficult to obtain the condition for the appearance of complex roots of this cubic equation. The condition has the form

$$\alpha^2p^2b > 4(1+\sqrt{5})^{-1}(3+\sqrt{5})^{-2} \simeq 0.09.$$

On the curve $b = 0.02$ (Fig. 1) this instability corresponds to segment 1.

Figure 2 presents the boundary of instability $k_z a = f(b)$ for $p = 1/43$. For small b it is determined by the condition $\operatorname{Re} z < \alpha$ and, approximately, $k_z a = b$; while for $b \gg 1$ the limiting value of $k_z a$ is determined by ion damping and proves to be almost independent of b. The dashed curve in Fig. 2 shows where the increment γ, for fixed values of b, reaches a maximum with respect to $k_z a$.

With the aid of the dispersion equation (3), one can investigate stability also in the more general case of a nonisothermal, anisotropic plasma, but this problem lies beyond the scope of the present article. We note only that, since the wave excitation investigated here is due to the electrons, the increment $\operatorname{Im}\omega$ is determined mainly by the electron temperature. In particular, the instability

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also occurs in the limiting case of cold ions $T_i \ll T_e$. It follows from this, in particular, that along with the effect of the finite Larmor radius, ion inertia also leads to a phase shift of the wave that promotes instability.

4. Thus, we have shown that an inhomogeneous isothermal plasma in a strong magnetic field is unstable with respect to the amplification of drift-type waves with transverse wavelength of the order of the mean Larmor radius of the ions, ρ_i. The corresponding perturbations are strongly elongated along the magnetic-field lines, and therefore the instability investigated here can actually appear only in such configurations whose longitudinal length L exceeds the transverse length a by an order of magnitude.

Since the transverse wavelength of the growing perturbations, for not too small values of a/L, is of the order of ρ_i, and the development time of the oscillations is determined by the quantity $a/\rho_i v_i$, where $v_i = \sqrt{T/m_i}$, one may think that the pulsations developing as a result of the instability will lead to diffusion of the plasma across the magnetic field with a diffusion coefficient of the order of $\rho_i^2 v_i/a$. This coefficient decreases as H^{-2} with increasing magnetic field, but in absolute magnitude it may considerably exceed the diffusion coefficient due to collisions.

The authors express their gratitude to E. P. Velikhov for discussion of the work.

Received
14 IV 1962

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