BIOPHYSICS

BIOCHEMISTRY

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Abstract

Full Text

LETTER TO THE EDITOR

To my article "On the completion and closure of an incomplete system of functions $\{e^{-\mu_k x} x^{s_k-1}\}$," published in DAN, vol. 141, No. 3, 1961, the following addition must be made.

Among other results, the article also contained a complete characterization of the subclass $M_2^{(\infty)}\{\mu_k\}$ of those functions from $L_2(0,+\infty)$ which admit approximation in L_2 by a system of functions of the form $\{e^{-\mu_k x} x^{s_k-1}\}$ in the case when $\operatorname{Re}\mu_k > 0$ and

$$\sum_{k=1}^{\infty} \frac{\operatorname{Re}\mu_k}{1+|\mu_k|^2} < +\infty. \qquad (*)$$

When submitting this article for publication I did not have at hand the book by L. Schwartz [1a] and cited it from an abstract in RZhMat [1b], where, as became clear later, the principal results of this book were not fully reflected.

For this reason, in my article it was stated that L. Schwartz's book gives only necessary conditions for a function $f(x)$ to belong to the class $M_2^{(\infty)}\{\mu_k\}$, whereas in fact it contains conditions of a necessary-and-sufficient type, characterizing functions of this class (and of more general classes $M_p^{(\infty)}\{\mu_k\}$) in the particular case when $\operatorname{Im}\mu_k = 0$, $s_k = 1$ $(k = 1, 2, ...)$.

At the same time, however, the following should be noted. The results contained in the article (the corollary to Theorem 2, Theorem 3 and its corollary), which

characterize the class $M_2^{(\infty)}\{\mu_k\}$, while differing in their form from the characterization given by L. Schwartz, are, of course, equivalent to it in substance in the particular case when $\operatorname{Im}\mu_k = 0$, $s_k = 1$ $(k = 1, 2, ...)$; but in the general case of complex exponents $\{\mu_k\}$ satisfying only condition $(*)$, they differ from it essentially. In the latter case the class $M_2^{(\infty)}\{\mu_k\}$ is substantially enlarged and may consist of functions which do not admit analytic continuation not only to the entire half-plane $\operatorname{Re} z > 0$, as is the case considered by L. Schwartz, but even to any angular domain $|\arg z| < \alpha < \pi/2$ containing the ray $(0, +\infty)$.

M. Dzhrbashyan

CITED LITERATURE

1. a) L. Schwartz, *Étude des sommes d' exponentielles*, 1st ed., 1943; 2nd ed., 1959; b) L. Schwartz, RZhMat., Abstract No. 8867 (1960).

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Note: Figure translations are in progress. See original paper for figures.