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SYNTHESIS OF ONE CLASS OF COMPUTING-AND-SOLVING DEVICES

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Abstract

Full Text

CYBERNETICS AND CONTROL THEORY

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SYNTHESIS OF ONE CLASS OF COMPUTING-AND-SOLVING DEVICES

(Presented by Academician V. A. Trapeznikov, 4 VI 1962)

In automatic control and monitoring systems there often arises the need to implement devices that carry out automatic tracking of one of the roots $x^*(t) = (x_1^*, \dots, x_n^*)$ of some system of finite equations

$$f_i(x_1, \dots, x_n, u_1, \dots, u_r) = 0 \qquad (i = 1, \dots, n) \tag{1}$$

with parameters $u_1, \dots, u_r$ that vary in time.

In most cases the variables $x_1, \dots, x_n$ are not expressed analytically through the independent variables, and therefore this problem is usually solved by using methods of analog computing-and-solving technology (1), which make it possible to carry out a functional transformation for functions $x_i = x_i(u_1, \dots, u_r)$ specified implicitly—by the system of equations (1).

Usually (1) a computing-and-solving device intended for generating implicit functions is constructed according to the principle of continuous working-off of the discrepancies $\xi_i = f_i$ as the variables $u_1, \dots, u_r$ change. Tracking a root by means of such a device is inevitably accompanied by an error in the steady-state regime. At appreciable rates of change of the variables $u_1, \dots, u_r$, the error may be significant, which under certain conditions causes bifurcation of the roots.

In the present note a method is set forth for synthesizing systems that make it possible to track roots in the steady-state regime with error equal to zero.

1°. Suppose that the functions f_i are continuously differentiable with respect to all their arguments in the domain D through which the solutions of the system of equations (1) pass, and that the functions $u_k(t)$ are bounded and differentiable with respect to t. In addition, the system of functions f_i $(i = 1, \dots, n)$ has everywhere in D a nonsingular Jacobi matrix $A = \{\partial f_i / \partial x_k\}$.

Consider the system of equations

$$\frac{df_i}{dt} = \Phi_i(f_1, \dots, f_n), \qquad i = 1, \dots, n, \tag{2}$$

where the functions Φ_i, as functions of the arguments $f_1, \dots, f_n$, satisfy the Lipschitz conditions everywhere and are chosen so that the system (2) in the phase space F, with coordinate axes $f_1, \dots, f_n$, has a unique asymptotically stable equilibrium point $\mathbf{f} = 0$ ($f_1 = f_2 = \dots = f_n = 0$). As such a system one may use, for example, the system

$$\frac{df_i}{dt} = -\lambda_i f_i, \qquad \lambda_i > 0, \qquad i = 1, \dots, n. \tag{3}$$

The system of equations (2) implicitly specifies a system of equations with respect to the variables $y_1, \dots, y_n$

$$\frac{dy}{dt} = \overline{A}^{1}\left(\Phi - B\frac{du}{dt}\right), \tag{4}$$

where dy/dt, du/dt, Φ are column matrices of derivatives and of the functions Φ_i, respectively; $\overline{A}^{1}$ is the matrix inverse to the Jacobi matrix; B is the $(n \times r)$ matrix of derivatives $\partial f_i/\partial u_k$. The system (2) has the following properties.

Every solution $x^*_k(t)$ (k is the number of the solution) of the system of equations (1) is a particular solution of system (4), for the reason that the point $\mathbf{f} = 0$ is an equilibrium point of system (2). Under the initial conditions $\mathbf{y}(t_0) = \mathbf{x}_k(t_0)$, satisfying equations (1), the solution $\mathbf{y}(t, \mathbf{y}(t_0))$ of the system of equations (4) for all $t > t_0$ will satisfy system (1), since for $t > t_0$, by virtue of (2),

$$f_i(\mathbf{y}(t, \mathbf{y}(t_0)), \mathbf{u}(t)) \equiv 0, \qquad i = 1, \dots, n.$$

If for $t = 0$ not all $\xi_i(t_0) = 0$,

$$\xi_i(t_0) = f_i(\mathbf{y}(t_0), \mathbf{u}(t_0)),$$

then, by virtue of asymptotic stability, for $t > t_0$ all ξ_i will tend to zero, and by virtue of the differentiability of the functions f_i and the boundedness of the functions $u_j(t)$, the estimate

$$\left(\sum_{i=1}^{n} (x^*_i(t) - y_i(t))^2\right)^{1/2} \leq C \max\{\xi_1, \dots, \xi_n\}, \qquad C = \text{const.}$$

follows.

It follows from the estimate that the phase trajectories $\mathbf{y}(t) \equiv \mathbf{x}_k(t)$ (k is the number of a root of the system of equations (1)) are asymptotically stable.

The dynamic error will decrease with time. In practice this means that, after the transient process has ended, the error in the steady-state regime will be equal to zero. At the same time, fixation of the root is achieved.

Equation (4) is implemented by means of analog technology, for example, by the solving elements of electronic models.

With the aid of system (4) one can also find the roots of finite equations (^{2}). In this case $u_j \equiv \text{const}$, $\dot{u}_j \equiv 0$, $j = 1, \ldots, n$. The equilibrium points of the equations

$$\frac{d\mathbf{y}}{dt} = \bar{A}^1 \Phi, \qquad \frac{d\mathbf{y}}{dt} = (\det A)^n C \Phi,$$

where C is the matrix adjoint to the matrix A, are asymptotically stable and coincide with the roots of the system of equations (1). When the initial conditions are specified in the domain of attraction of some root, the representing point will subsequently tend to this root.

2°. Consider the matrix equation:

$$\frac{d\mathbf{f}}{dt} = 0.$$

The equilibrium point $\mathbf{f} = 0$ of this equation is Lyapunov stable; accordingly, the trajectories $\mathbf{y}(t) \equiv \mathbf{x}_k^*(t)$ of the equation

$$\frac{d\mathbf{y}}{dt} = -\bar{A}^1 B \frac{d\mathbf{u}}{dt}. \tag{5}$$

are also Lyapunov stable.

A computing-solving device with equation of motion (5) makes it possible to track a root with an error not exceeding ε. Indeed, if the trajectory is Lyapunov stable, then always, for a given $\varepsilon > 0$, there exists a $\delta(\varepsilon) > 0$ such that, if the error in the initial conditions

$$\delta_0 = \left(\sum_{i=1}^{n} (x_i^*(t_0) - y_i(t_0)) \right)^{1/2}$$

does not exceed δ, then for $t > t_0$ the error will not exceed the prescribed number ε.

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.

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