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Abstract

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ON THE PROBLEM OF CONSTRUCTING HIGH-QUALITY AUTOMATIC CONTROL SYSTEMS FOR PLANTS WITH VARYING PARAMETERS

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In a number of cases it becomes necessary to construct automatic control systems for plants whose parameters vary within fairly wide limits. In this paper an attempt is made to solve this problem by using certain properties of automatic control systems with variable structure ([1]). The possibility is considered of constructing, by means of a certain fixed piecewise-linear control law, such automatic control systems in which the quality of the transient processes, when the plant parameters vary within wide limits, changes only slightly (within admissible limits).

Let, in a domain G of the n-dimensional space $x_1, \dots, x_n$, the motion of a dynamical system be described by the system of differential equations

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}), \tag{1}$$

where $\mathbf{x} = (x_1, \dots, x_n)$; $\mathbf{f} = (f_1, \dots, f_n)$; $f_i = x_{i+1}$ $(i = 1, 2, \dots, n-1)$;

$$f_n = \sum_{i=1}^{n} \psi_i(\mathbf{x}) x_i;$$

$$\psi_i(\mathbf{x}) = \begin{cases} a_i, & \text{when } \sum_{j=1}^{n} c_{jx} j > 0, \\ b_i, & \text{when } \sum_{j=1}^{n} c_{jx} j < 0 \end{cases} \qquad (i = 1, 2, \dots, n);$$

a_i, b_i, c_j are constant quantities.

Let the hyperplane S, defined by the equation $\sum_{j=1}^{n} c_{jx}j = 0$, divide the domain G into the domains G^+ $\left(\sum_{j=1}^{n} c_{jx}j > 0\right)$ and G^- $\left(\sum_{j=1}^{n} c_{jx}j < 0\right)$, in which the vector function $\mathbf{f}(\mathbf{x})$ of system (1) is bounded, and for any constant value of time t, as S is approached from G^+ and G^-, there exist its limiting values $\mathbf{f}^+$ and $\mathbf{f}^-$. Suppose that, as the solution $\mathbf{x}(t)$ approaches some domain $U \subset S$, the vector functions $\mathbf{f}^+$ and $\mathbf{f}^-$ are directed toward the hyperplane S ($f_N^+(\mathbf{x}) \leq 0$,

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* In the case $\sum_{j=1}^{n} c_{jx}j = 0$

$$\psi_i(\mathbf{x}) = a_i \quad \text{when} \sum_{j=1}^{n} c_{jx}j \to +0, \qquad \psi_i(\mathbf{x}) = b_i \quad \text{when} \sum_{j=1}^{n} c_{jx}j \to -0.$$

$f_N^+(0) \leqslant 0,\ f_N^- - f_N^+ > 0$, where f_N^+ and f_N^- are the projections of the vectors $\mathbf{f}^+$ and $\mathbf{f}^-$ onto the normal to the hyperplane S, directed from G^- to G^+). Then, when $\mathbf{x}(t)$ enters U, the so-called "sliding mode" arises, and the solution of system (1) does not depend on a_i and b_i.

Indeed, in this case, as shown in the work of A. F. Filippov (2), in the domain U there exists a solution $\mathbf{x}(t)$ of system (1), and the vector $d\mathbf{x}/dt = \mathbf{f}^0(\mathbf{x})$ lies in the hyperplane S and is determined by the values of the vector-functions $\mathbf{f}^+$ and $\mathbf{f}^-$.

From the condition that $\mathbf{f}^0 \in S$, there follows the linear dependence of the components of the vector $\mathbf{f}^0$

$$\sum_{j=1}^{n} c_j f_j^0 = 0, \tag{2}$$

where f_j^0 is the j-th component of the vector $\mathbf{f}^0$, whence

$$f_n^0 = \frac{1}{c_n} \sum_{j=1}^{n-1} c_j f_j^0. \tag{3}$$

Consequently, the solution of system (1) for $\mathbf{x}(t) \in U$ coincides with the solution of the linear system of differential equations

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}^0(\mathbf{x}), \tag{4}$$

where $\mathbf{x} = (x_1, \dots, x_n)$; $\mathbf{f}^0 = (f_1^0, \dots, f_n^0)$; $f_j^0 = x_{j+1}$ $(j = 1, 2, \dots, n-1)$;

Fig. 1

Figure 1: Fig. 1

$$f_n^0 = \frac{1}{c_n} \sum_{j=1}^{n-1} c_j x_j; \quad c_j$$

are constant quantities.

It is evident that the solution of system (4) does not depend on a_i and b_i. Let the parameters of the controlled plant that determine the coefficients a_i and b_i of the system of equations (1) vary in such a way that, during the transient process, a_i and b_i may be regarded as constant. We shall construct the automatic control system in such a way that, when these parameters vary within sufficiently wide limits: 1) the domain U exists, includes the origin, and the solution of the system of differential equations (4) satisfies the prescribed requirements for the quality of the control process (the settling time and the maximum dynamic error must not exceed previously assigned values); 2) for any initial conditions the solution of the system of equations (1) enters the domain U, and the coordinate x_1 changes sign no more than once.

Then the solution of the original system (1) will depend on the parameters of the plant only up to the moment when $\mathbf{x}(t)$ enters the domain U, where the solution depends only on the coefficients c_j.

Consequently, the influence of changes in the plant parameters on the quality of the control process is substantially weakened. Physically, this effect can be explained by means of analogies between relay systems operating in a sliding mode and systems having an infinite gain, considered by Ya. Z. Tsypkin (3). Indeed, for $\mathbf{x}(t) \in U$,

$$\sum_{j=1}^{n} c_j x_j$$

is the input quantity of the element implementing the function $\psi_i(\mathbf{x})$, and is close to zero, whereas its output quantity $x(t)$ in the nonstationary mode is different from zero. Consequently, in the sliding mode this element may be regarded as an amplifying link with an infinite gain.

Thus, the solution of the problem reduces to choosing a control law satisfying the requirements stated above, i.e., to completing the definition of the function $\psi_i(\mathbf{x})$.

Example. Let the following system (Fig. 1) be described by the system of differential equations

Fig. 1

Fig. 2

Figure 2: Fig. 2

$$\frac{dx_1}{dt} = x_2, \qquad \frac{dx_2}{dt} = \psi(x_1, x_2)x_1, \tag{5}$$

where

$$\psi(x_1, x_2) = \begin{cases} a_1, & \text{for } \Phi(x_1, x_2) > 0, \\ b_1, & \text{for } \Phi(x_1, x_2) < 0^*; \end{cases}$$

$$\Phi(x_1, x_2) = x_1(c_1x_1 + c_2x_2); \qquad a_1 = +\omega_0^2; \qquad b_1 = -\omega_0^2; \qquad \omega_0^2 = \frac{K_{sK}a}{T_s}; \qquad K_a,$$

K_s, T_s, c_1, c_2 are constants.

Let the plant parameter K_a vary within the limits from $K_{a\,\min}$ to $K_{a\,\max} \to \infty$. It is required to choose such values of K_s/T_s and c_1/c_2 that, when K_a

Fig. 2. a—$\omega_0 = c_1/c_2$; b—ω_0'; v—ω_0''; $\omega_0 < \omega_0' < \omega_0''$

varies within the specified limits, the transient processes in the tracking system proceed without overshoot and the tracking time t_{track} does not exceed the prescribed value.

On the basis of an analysis of the phase portrait of the system (Fig. 2), taking into account that the application of a step-like control action $g(t)$ is equivalent in this case to nonzero initial conditions in x_1, Fig. 3 gives plots of the dependence of t_{track} on ω_0. By tracking time is meant the time over which the dynamic error x_1 decreases to a value equal to 5% of $x_{1\,\max}$. The dashed curve in Fig. 3 corresponds to the dependence of t_{track} on ω_0 for $c_1/c_2 = \omega_0$. The solid line represents the dependence of t_{track} on ω_0 for a constant controller setting $c_1/c_2 = \text{const}$ and variation of ω_0 from $\omega_0 = c_1/c_2$ to $\omega_0 \to \infty$.

* In the case $\Phi(x_1, x_2) = 0$

$$\psi(x_1, x_2) = a_1 \quad \text{for } \Phi(x_1, x_2) \to +0,$$

$$\psi(x_1, x_2) = b_1 \quad \text{for } \Phi(x_1, x_2) \to -0.$$

The dependence of t_{track} on ω_0 for $c_1/c_2 = \text{const}$ in Fig. 3 was obtained under the assumption that ω_0 changes due to a change in K_a within the limits from $K_{a\,\min}$ to $K_{a\,\max}$.

Fig. 3

Figure 3: Fig. 3

Let K_a vary within the limits from 1 to 1000. It is required to construct a system in which the transient processes occur without overshoot and $t_{\text{track}} \leqslant 2$ sec.

Fig. 3

From the graph in Fig. 3, for the given value $t_{\text{track}} = 2$ sec, we determine the value

$$c_1/c_2 = \omega_0 = K_{a\,\min} K_s/T_s = 1.6,$$

corresponding to the value $K_{a\,\min} = 1$. Hence we find the required value $K_s/T_s = 1.6$. When K_a changes from 1 to 1000, the transient process proceeds without overshoot, and t_{track} decreases (Fig. 3).

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Note: Figure translations are in progress. See original paper for figures.