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Abstract

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ON THE INTERACTION OF CHARGES WITH AN ELECTRON PLASMA IN A MAGNETIC FIELD

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The radiation of charges passing through an electron plasma in an external magnetic field has been studied in a number of works ([1,2,9]). In the present paper the energy losses of a charged particle moving along the external magnetic field $\mathbf{H}_0$ are determined. It is assumed that the velocity of the particle is considerably greater than the thermal velocity of the plasma electrons. In limiting cases, previously known results ([2]) are obtained. Coherent radiation arising when a spherically symmetric bunch passes through a magnetoactive plasma is also considered. We note that the influence of the magnetic field on Coulomb collisions of charged plasma particles having velocities of the order of the mean thermal velocity was investigated by S. T. Belyaev ([3]) and V. P. Silin ([4]).

For the energy losses per unit path length we have

$$-\frac{dE}{dz} = -\int \frac{\rho}{v}(\mathbf{vE})\,d\mathbf{r}, \tag{1}$$

where ρ is the charge density, $\mathbf{v}$ is the velocity of its motion, and $\mathbf{E}$ is the electric field produced by the charge in the plasma. The function ρ for a point charge has the form

$$\rho = q\delta(\mathbf{r} - \mathbf{v}t). \tag{2}$$

For a bunch we choose the charge-density distribution in the form

$$\rho = q\frac{\alpha^2}{4\pi}e^{-\alpha|\mathbf{r}-\mathbf{v}t|}, \tag{3}$$

where q is the total charge of the bunch, and the parameter α characterizes the dimensions of the bunch.

The field $\mathbf{E}$ can be determined by using Maxwell' s equations and the equations of motion of the electron "fluid." In doing so, we introduce into the equations of motion a friction term that provides a small damping of the waves. Quantities depending on the coordinate $\mathbf{r}$ and time t are expanded in plane waves $\exp i(\mathbf{kr}-\omega t)$. We represent the dielectric-permittivity tensor in the form

$$\varepsilon_{ij} = a\delta_{ij} + b\varepsilon_{ijl}h_l + dh_i h_j, \tag{4}$$

where $\mathbf{h}$ is a unit vector in the direction of the external magnetic field, and ε_{ijl} is the antisymmetric tensor of rank three. The coefficients a, b, d are equal to

$$a = 1 - \frac{\omega'}{\omega}\frac{\Omega^2}{\omega a'^2 - \omega_H^2}, \qquad b = -i\frac{\Omega^2\omega_H}{\omega(\omega a'^2 - \omega_H^2)}, \qquad d = \frac{\Omega^2\omega_H^2}{\omega\omega'(\omega a'^2 - \omega_H^2)}, \tag{5}$$

where Ω is the Langmuir frequency, ω_H is the gyrofrequency of the electrons, $\omega' = \omega + i\nu$, and ν is the frequency of "close" collisions. We shall regard the frequency ν as small in comparison with the frequency of the emitted waves. The introduction of ν makes it possible to establish the direction in which the poles of the integrand in expression (1) are to be bypassed.

For the energy losses per unit path we obtain

$$-\frac{dE}{dz} = -\frac{q^2 i}{\pi}\iint \frac{\Phi^2(k)\left[-n^4\chi_3^2 + n^2 a(1+\chi_3^2) - a^2 - b^2\right]\, k\,dk\,d\omega}{\omega\{(a+d\chi_3^2)n^4 + n^2[-2a^2 - b^2 - ad + (b^2 - ad)\chi_3^2] + (a+d)(a^2+b^2)\}}, \tag{6}$$

where k is the magnitude of the wave vector, $n = kc/\omega$, $\chi_3 = \omega/kv$. The function $\Phi(k)$, in the case of an arbitrary spherically symmetric charge distribution, is determined by the relation

$$\Phi(k) = \frac{4\pi}{k}\int_0^\infty \sin kR \cdot \rho(R)\, R\, dR. \tag{7}$$

For a point charge the function $\Phi(k)$ is equal to 1, while for a bunch with charge density (3) we find

$$\Phi_{\mathrm{b}}(k) = \frac{\alpha^4}{(k^2+\alpha^2)^2}. \tag{8}$$

The integration over k in formula (6) is carried out from $|\omega/v|$ to k_m. The introduction of the parameter k_m is connected with the fact that the macroscopic description of collisions becomes inapplicable for small collision parameters. Performing the integration over k, we obtain the following expression for the energy losses of a point charge per unit path:

$$-\left(\frac{dE}{dz}\right)_j = \frac{q^2}{4v^2}\int\left[-(1-\beta^2)-\frac{\Omega^2}{\omega^2-\omega_H^2-\Omega^2}-(-1)^j\frac{(1-\beta^2)^2+\dfrac{\Omega^2(1+\beta^2)}{\omega^2-\omega_H^2-\Omega^2}}{\left[(1-\beta^2)^2+4\beta^2\dfrac{\omega^2-\Omega^2}{\omega_H^2}\right]^{1/2}}\right]\delta_j(\omega^2)\,d\omega^2, \tag{9}$$

where $\beta = v/c$, and j is the wave number ($j = 1, 2$). The function $\delta_j(\omega^2)$ is equal to $+1$ or -1 depending on the direction in which the singular point is encircled in the integration in the k-plane. Using expressions (4) and (5), one can obtain

$$\delta_j = \operatorname{sgn}\left\{(-1)^j\left[(1+\beta^2)q+2\beta^2(q+p)p\right]+\left[(1+\beta^2)q+2\beta^2p^2\right]\sqrt{(1+\beta^2)^2+4\beta^2p}\right\}, \tag{10}$$

where

$$q = \frac{3\omega^2-2\Omega^2-\omega_H^2}{\omega_H^2}, \qquad p = \frac{\omega^2-\omega_H^2-\Omega^2}{\omega_H^2}. \tag{10a}$$

The integration in expression (8) is carried out over the frequency regions for which the inequalities

$$\frac{k_m^2v^2}{\omega^2} \gg \beta^2 n_j^2 \gg 1, \tag{11}$$

are satisfied, where the refractive indices $n_{1,2}$ are determined by the expression

$$n_j^2 = \frac{2\beta^2(\omega^2-\Omega^2)(\omega^2-\omega_H^2-\Omega^2)-\Omega^2\omega_H^2(1+\beta^2)-(-1)^j\omega_H\sqrt{(1-\beta^2)^2\omega_H^2+4\beta^2(\omega^2-\Omega^2)}}{2\beta^2\omega^2(\omega^2-\omega_H^2-\Omega^2)}. \tag{12}$$

The frequency intervals for which the condition $\beta^2 n_j^2 \gg 1$ is satisfied, and the corresponding values of $\delta_j(\omega^2)$, are given in Table 1, where

$$\omega_{1,2}^2 = \frac{(1-\beta^2)\omega_H^2-2\beta^2\Omega^2\pm\omega_H\sqrt{(1-\beta^2)^2\omega_H^2-4\beta^2(1-\beta^2)\Omega^2}}{2(1-\beta^2)}; \tag{13}$$

$$\omega_3^2 = \begin{cases}\Omega^2-\dfrac{(1-\beta^2)^2}{4\beta^2}\,\omega_H^2, & \text{for } \dfrac{\omega_H^2}{\Omega^2} \ll \dfrac{4\beta^2}{(1-\beta^2)^2},\\[2ex] 0, & \text{for } \dfrac{\omega_H^2}{\Omega^2} \gg \dfrac{4\beta^2}{(1-\beta^2)^2}.\end{cases} \tag{14}$$

Let us note that the values $\delta_j(\omega^2)$ indicated in the table do not depend on precisely which mechanism leads to the small damping of the waves. Consequently, in order to determine $\delta_j(\omega^2)$ it is not necessary to use the tensor ε_{ij} in the form (4)–(5).

Table 1

Relation between the plasma parameters and the particle velocity	Wave number j	Frequency intervals of the emitted waves	$\delta_j(\omega^2)$
$\frac{\omega_H^2}{\Omega^2} \ll \frac{4\beta^2}{1-\beta^2}$	2	$\left[\Omega, \sqrt{\Omega^2+\omega_H^2}\right]$	1
$\frac{4\beta^2}{1-\beta^2} \ll \frac{\omega_H^2}{\Omega^2} \ll \frac{1}{(1-\beta^2)^2}$; $\beta^2 \geq \frac{1}{2}$	2	$\begin{cases}[\Omega, \omega_2] \\ \left[\omega_1, \sqrt{\Omega^2+\omega_H^2}\right]\end{cases}$	1 1
$\frac{4\beta^2}{1-\beta^2} \ll \frac{\omega_H^2}{\Omega^2} \ll \min\left\{\frac{1}{(1-\beta^2)^2}, \frac{4\beta^2}{(1-\beta^2)^2(1+\beta^2)}\right\}$; $\beta^2 \ll \frac{1}{2}$	2	$\begin{cases}[\omega_2, \omega_1] \\ \left[\Omega, \sqrt{\Omega^2+\omega_H^2}\right]\end{cases}$	−1 1
$\frac{1}{(1-\beta^2)^2} \ll \frac{\omega_H^2}{\Omega^2} \ll \frac{4\beta^2}{(1-\beta^2)^2(1+\beta^2)}$; $\beta^2 \geq \frac{1}{3}$	2	$\begin{cases}[\omega_2, \Omega] \\ \left[\omega_1, \sqrt{\Omega^2+\omega_H^2}\right]\end{cases}$	−1 1

Relation between the plasma parameters and the particle velocity	Wave number j	Frequency intervals of the emitted waves	$\delta_j(\omega^2)$
$\frac{4\beta^2}{(1-\beta^2)^2(1+\beta^2)} \ll \frac{\omega_H^2}{\Omega^2} \ll \frac{1}{(1-\beta^2)^2}$; $\beta^2 \ll \frac{1}{3}$	$\begin{cases} 2 \\ 1 \end{cases}$	$\begin{cases} [\omega_3, \omega_1] \\ \left[\Omega, \sqrt{\Omega^2+\omega_H^2}\right] \\ [\omega_3, \omega_2] \end{cases}$	-1 1 1 -1
$\frac{\omega_H^2}{\Omega^2} \gg \max\left\{\frac{1}{(1-\beta^2)^2}, \frac{4\beta^2}{(1-\beta^2)^2(1+\beta^2)}\right\}$	$\begin{cases} 2 \\ 1 \end{cases}$	$\begin{cases} [\omega_3, \Omega] \\ \left[\omega_1, \sqrt{\Omega^2+\omega_H^2}\right] \\ [\omega_3, \omega_2] \end{cases}$	1 1 1

As is seen from the table, in some cases the frequency regions of the emitted ordinary and extraordinary waves overlap. For example, for

$$\frac{\omega_H^2}{\Omega^2} > \frac{4\beta^2}{(1-\beta^2)^2}$$

emission of both ordinary and extraordinary waves is possible in the frequency region close to zero.

In order to obtain the total energy losses of a point charge, one must add to the losses calculated by formula (9) the energy losses in "close" collisions. It should be borne in mind that the magnetic field has little effect on the magnitude of the energy losses in "close" collisions. Therefore the energy loss of a point charge per unit path can be represented in the form

$$-\left(\frac{dE}{dz}\right)_{H_0\neq 0} = -\left(\frac{dE}{dz}\right)_{H_0=0} + F(\omega_H). \tag{15}$$

The energy losses in the absence of an external magnetic field were determined in papers ([5–8]). Assuming $k_m v \gg \Omega,\ \omega_H$, for $F(\omega_H)$ we obtain the follow-

expressions. If $\omega_H^2/\Omega^2 \ll 4\beta^2/(1-\beta^2)$, then

$$F(\omega_H) = -\frac{q^2\omega_H^2}{4v^2}(1-\beta^2)(2-\beta^2). \tag{16}$$

In the case $4\beta^2/(1-\beta)^2 \ll \omega_H^2/\Omega^2 \ll 4\beta^2/(1-\beta^2)^2$, we have

$$F(\omega_H) = \frac{q^2\Omega^2}{2v^2}\left[\ln\frac{1-x}{1+x} - \frac{(1-\beta^2)(2-\beta^2)\omega_H^2}{2\Omega^2} + \frac{(1-\beta^2)\omega_H^2 x}{2\beta^2\Omega^2}\right], \tag{17}$$

where

$$x = \sqrt{1-\frac{4\beta^2\Omega^2}{(1-\beta^2)\omega_H^2}}. \tag{17a}$$

And, finally, for $\omega_H^2/\Omega^2 \gg 4\beta^2/(1-\beta^2)^2$, we find

$$F(\omega_H) = \frac{q^2\Omega^2}{2v^2}\left\{\ln\frac{(1-x)(1+\beta^2+y)}{(1+x)(1+\beta^2-y)} - \frac{(1-\beta^2)\omega_H^2}{2\beta^2\Omega^2}\left[\beta^2(2-\beta^2)-x+(1-\beta^2)y\right]\right\}, \tag{18}$$

where

$$y = \sqrt{(1-\beta^2)^2 - 4\beta^2\frac{\Omega^2}{\omega_H^2}}. \tag{18a}$$

From formulas (15)–(18) follow the expressions given in paper (2) for the energy losses in the nonrelativistic ($\beta \to 0$) and relativistic ($\beta \to 1$) cases, as well as in the case of a "strong" magnetic field ($(1-\beta^2)\omega_H \gg \Omega$). In the nonrelativistic case, the energy losses of a charged particle moving along the external magnetic field were found by means of the methods of quantum field theory in paper (9).

Thus, a charge passing through a plasma in an external magnetic field emits ordinary and extraordinary waves. This radiation may escape from the plasma. The magnitude of the radiation losses is comparable with the magnitude of the losses due to "close" Coulomb collisions. The frequencies of the emitted waves are given in Table 1. The intensity of radiation of ordinary and extraordinary waves in any frequency interval is easily estimated from formula (9).

Using the data presented in the table, one may also calculate the energy losses for a bunch with charge density determined by expression (3). If the inequalities $\alpha v \gg \Omega, \omega_H$ are satisfied, the energy losses of the bunch are coherent*. In this case we obtain

$$-\left(\frac{dE}{dz}\right)_{\text{cr}} = \frac{q^2\Omega^2}{v^2}\ln\frac{0.4\alpha v}{\Omega} + F(\omega_H). \tag{19}$$

The radiation intensity is proportional to the square of the total charge of the bunch, i.e., to the square of the total number of charged particles in the bunch. The energy losses due to "close" collisions, proportional to the first power of the number of particles in the bunch, are small in comparison with the radiation losses. Naturally, in our treatment the deformation of the bunch during its motion is not taken into account.

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* Coherent radiation of a bunch and of systems of bunches in isotropic media was investigated by B. M. Bolotovskii.

Note: Figure translations are in progress. See original paper for figures.