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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

MECHANICS

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EQUATIONS OF MOTION OF THE BASIC BODY OF A SYSTEM OF RIGID BODIES OF VARIABLE COMPOSITION

(Presented by Academician I. I. Artobolevskii on 6 IV 1961)

A rigid body of variable composition is a body of variable composition ⁽¹⁾ in which the change of composition can occur only through the separation of material particles from the instantaneous outer surface of this body; moreover, the aggregate of particles that at each given instant of time are located in the region bounded by the outer surface of the body is a rigid body. By the instantaneous outer surface of a body of variable composition is meant the outer surface of the rigid body that at the given instant coincides with the body of variable composition.

Consider a system G of a finite number of rigid bodies of variable composition (shown schematically in Fig. 1), formed by attaching to the basic body G_0 s kinematic chains in such a way that in each chain every body $G_\nu^{(\sigma)}$ ($\nu = 1, 2, \dots, n_\sigma$; $\sigma = 1, 2, \dots, s$) has up to 6 degrees of freedom inclusive with respect to the preceding body $G_{\nu-1}^{(\sigma)}$; by the body $G_0^{(\sigma)}$ the body G_0 is meant. It is assumed that each of the bodies of the system G contains a part that remains unchanged during the entire time $0 \leq t \leq T$ ($0 < T \leq \infty$) of consideration of this system, and that the constraints in the system G are imposed on the aforementioned unchanged parts of the bodies $G_0, G_\nu^{(\sigma)}$. Some systems of type G were considered in works ⁽²⁻⁵⁾; a general theory of a system of bodies of arbitrary form was presented in the report by A. I. Lur' e "Problems in the Theory of Relative Motion" at the First Congress on Mechanics (January 1960) ⁽⁶⁾. Here the equations of motion of the body G_0 are given for prescribed laws of motion of all bodies $G_\nu^{(\sigma)}$ relative to the bodies $G_{\nu-1}^{(\sigma)}$ and for prescribed external and reactive forces.

Fig. 1

Introduce the notation: $N_\lambda^{(\sigma)} x_\lambda^{(\sigma)} y_\lambda^{(\sigma)} z_\lambda^{(\sigma)}$ ($\lambda = 0, 1, 2, \dots, n_\sigma$) is a system (σ) of

coordinates (rectangular, right-handed) connected with the unchanged part of the body $G_\lambda^{(\sigma)}$; $c_\lambda^{(\sigma)}$ is the row matrix of the coordinates of the center of mass of the body $G_\lambda^{(\sigma)}$ in the system (σ) ; $m_\lambda^{(\sigma)}$ is the mass of the body $G_\lambda^{(\sigma)}$; $I_\lambda^{(\sigma)}$ is the matrix of the moments of inertia (denoted in the usual way ⁽⁷⁾) of the body $G_\lambda^{(\sigma)}$ with respect to the axes of the system (λ) :

$$I_\lambda^{(\sigma)} = \left\| \begin{array}{ccc} A_\lambda^{(\sigma)} & -F_\lambda^{(\sigma)} & -E_\lambda^{(\sigma)} \\ -F_\lambda^{(\sigma)} & B_\lambda^{(\sigma)} & -D_\lambda^{(\sigma)} \\ -E_\lambda^{(\sigma)} & -D_\lambda^{(\sigma)} & C_\lambda^{(\sigma)} \end{array} \right\|, \quad k_\nu^{(\sigma)} = \frac{1}{2} (A_\nu^{(\sigma)} + B_\nu^{(\sigma)} + C_\nu^{(\sigma)}),$$

m is the mass of the system G ; $l_{pq}^{(\sigma)} = \|l_{ij}^{(pq,\sigma)}\|$ ($i, j = 1, 2, 3$; $p, q = 0, 1, 2, \dots, n_\sigma$; $l_{12}^{(pq,\sigma)} = \cos(x_p^{(\sigma)}, y_q^{(\sigma)})$) is the matrix of direction cosines between the axes of the systems (p) and (q) ; $h_{\nu-1}^{(\sigma)}$ is the row matrix of the coordinates of the origin $N_\nu^{(\sigma)}$ in the system $(\nu-1)$; $\omega_{pq}^{(\sigma)}$ is the row matrix of the projections of the instantaneous angular velocity of the system (p) relative to the system (q) , taken on the axes of the system (σ) ; $V_\lambda^{(\sigma)}$ is the row matrix of the projections of the principal vector of forces, external with respect to the system G and applied to the body $G_\lambda^{(\sigma)}$, as well as of the reactive forces applied to this body, taken on the axes of the system (λ) ; $M_\lambda^{(\sigma)}$ is the row matrix of the projections of the principal moment of the aforementioned forces with respect to the origin $N_\lambda^{(\sigma)}$, taken on the axes of the system (λ) ; \sim is the operator that transforms the row matrix $a = \|a_1, a_2, a_3\|$ into the matrix \tilde{a} (8)

$$\tilde{a} = \left\| \begin{array}{ccc} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{array} \right\|;$$

E is the unit matrix (3×3); a prime denotes matrix transposition. The masses $m_\lambda^{(\sigma)}$ and the moments of inertia $A_\lambda^{(\sigma)}, \dots, F_\lambda^{(\sigma)}$ are, generally speaking, nonincreasing functions of time; the elements of the matrices $c_\lambda^{(\sigma)}, h_{\nu-1}^{(\sigma)}$, and $l_{\nu(\nu-1)}^{(\sigma)}$ are functions of time. Denote by v the row matrix of the projections of the absolute velocity of the origin N_0 on the axes of the system $N_0x_0y_0z_0$, associated with the invariable part of the principal body; by ω , the row matrix of the projections of the absolute angular velocity of the system $x_0y_0z_0$ on the same axes, and

$$S_\nu^{(\sigma)} = \sum_{\mu=1}^{\nu} h_{\mu-1}^{(\sigma)} l_{(\mu-1)0}^{(\sigma)}.$$

The motion of the origin N_0 is described by the equations

$$\begin{aligned} \frac{dv}{dt} = v\tilde{\omega} + \frac{1}{m} \sum_{\sigma=1}^s \sum_{\nu=1}^{n_\sigma} \{ & V_0 - m_0 c_0 + V_\nu^{(\sigma)} l_{\nu 0}^{(\sigma)} + m_\nu^{(\sigma)} [2(c_\nu^{(\sigma)} \dot{l}_{\nu 0}^{(\sigma)} + \dot{S}_\nu^{(\sigma)}) \tilde{\omega} \\ & - (S_\nu^{(\sigma)} + c_\nu^{(\sigma)} l_{\nu 0}^{(\sigma)}) (\tilde{\omega}^2 - \dot{\tilde{\omega}}) - (c_\nu^{(\sigma)} \ddot{l}_{\nu 0}^{(\sigma)} + \ddot{S}_\nu^{(\sigma)})] \}, \end{aligned} \quad (1)$$

and the motion of the system $x_0 y_0 z_0$ about the origin N_0 is described by the equations

$$\begin{aligned} & \frac{d\omega}{dt} \sum_{\sigma=1}^s \sum_{\nu=1}^{n_\sigma} \{ I_0 + l_{\nu 0}^{(\sigma)'} I_\nu^{(\sigma)} l_{\nu 0}^{(\sigma)} - m_\nu^{(\sigma)} [(\tilde{S}_\nu^{(\sigma)})^2 + U_\nu^{(\sigma)} + U_\nu^{(\sigma)'}] \} = \\ = & M_0 + \omega I_0 \tilde{\omega}' + (\dot{v} - v\tilde{\omega}) \left[m_0 \tilde{c}_0 + \sum_{\sigma=1}^s \sum_{\nu=1}^{n_\sigma} m_\nu^{(\sigma)} (\tilde{S}_\nu^{(\sigma)} + l_{\nu 0}^{(\sigma)'} \tilde{c}_\nu^{(\sigma)} l_{\nu 0}^{(\sigma)}) \right] + \\ & + \sum_{\sigma=1}^s \sum_{\nu=1}^{n_\sigma} \{ m_\nu^{(\sigma)} (\ddot{S}_\nu^{(\sigma)} - 2\dot{S}_\nu^{(\sigma)} \tilde{\omega} + S_\nu^{(\sigma)} \tilde{\omega}^2) (\tilde{S}_\nu^{(\sigma)} + l_{\nu 0}^{(\sigma)'} \tilde{c}_\nu^{(\sigma)} l_{\nu 0}^{(\sigma)}) + \\ & + m_\nu^{(\sigma)} c_\nu^{(\sigma)} [(\tilde{\omega}_{\nu 0}^{(\sigma)})^2 - \dot{\tilde{\omega}}_{\nu 0}^{(\sigma)} + 2\tilde{\omega}_{\nu 0}^{(\sigma)} l_{\nu 0}^{(\sigma)} \tilde{\omega}_{\nu 0}^{(\sigma)'} + l_{\nu 0}^{(\sigma)} \tilde{\omega}^2 l_{\nu 0}^{(\sigma)'}] l_{\nu 0}^{(\sigma)} \tilde{S}_\nu^{(\sigma)} + \\ & + [\omega_{\nu 0} l_{\nu 0}^{(\sigma)} \tilde{\omega}_{\nu 0}^{(\sigma)} - \dot{\omega}_{\nu 0}^{(\sigma)} I_\nu^{(\sigma)} + 2\omega l_{\nu 0}^{(\sigma)'} (I_\nu^{(\sigma)} - k_\nu^{(\sigma)} E) \tilde{\omega}_{\nu 0}^{(\sigma)}] l_{\nu 0}^{(\sigma)} \\ & - \omega l_{\nu 0}^{(\sigma)'} I_\nu^{(\sigma)} l_{\nu 0}^{(\sigma)} \tilde{\omega} - M_\nu^{(\sigma)} l_{\nu 0}^{(\sigma)} - V_\nu^{(\sigma)} l_{\nu 0}^{(\sigma)} \sum_{\kappa=1}^{\nu} l_{(\kappa-1)0}^{(\sigma)'} \tilde{h}_{\kappa-1}^{(\sigma)} l_{(\kappa-1)0}^{(\sigma)} \}, \end{aligned} \quad (2)$$

where

$$U_\nu^{(\sigma)} = \tilde{S}_\nu^{(\sigma)} l_{\nu 0}^{(\sigma)'} \tilde{c}_\nu^{(\sigma)} l_{\nu 0}^{(\sigma)}.$$

In this case,

$$j_{x0}^{(\sigma)} = -\tilde{\omega}_{x0}^{(\sigma)} l_{x0}^{(\sigma)}, \quad \ddot{l}_{x0}^{(\sigma)} = \left[(\tilde{\omega}_{x0}^{(\sigma)})^2 - \dot{\tilde{\omega}}_{x0}^{(\sigma)} \right] l_{x0}^{(\sigma)},$$

$$l_{x0}^{(\sigma)} = \prod_{\alpha=1}^x l_{(x-\alpha+1)(x-\alpha)}^{(\sigma)}, \quad \omega_{x0}^{(\sigma)} = \sum_{\beta=0}^{x-1} \omega_{(x-\beta)(x-\beta-1)}^{(\sigma)} l_{(x-\beta)0}^{(\sigma)}.$$

The derivative \dot{v} also enters the right-hand side of equation (2), the derivative $\dot{\omega}$ enters the right-hand side of equation (1), and the matrix of moments of inertia of the system G enters the left-hand side of equation (2) as a multiplier of the derivative $\dot{\omega}$. However, in computations with the aid of computing devices this circumstance is not restrictive. In principle, the system of equations (1) and (2) is solvable with respect to the derivatives \dot{v} and $\dot{\omega}$, but the form of the right-hand sides thereby becomes unnecessarily complicated.

Equations (1) and (2) may also describe the relative motion of any one of the bodies $G_\nu^{(\sigma)}$, if the absolute motion of the body G_0 and the relative motions of all the remaining bodies of the system G are prescribed. Equations (1) and (2) are also applicable to a system G^* of more complex structure, differing from the system G by the presence of parallel chains of bodies (still open), branching from all or some of the bodies $G_\nu^{(\sigma)}$. The system G^* is reduced to the system G by introducing additional chains, branching from the body G_0 and regarded as massless on the segments from the body G_0 to the corresponding parallel branch.

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Note: Figure translations are in progress. See original paper for figures.

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