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Abstract

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ANALYTIC PROPERTIES OF THE ELASTIC $\pi\pi$-SCATTERING AMPLITUDE IN THE l-PLANE

(Presented by Academician N. N. Bogolyubov, 2 IV 1962)

1. We recently showed that, if the partial amplitude of $\pi\pi$ scattering is meromorphic in l, then the results of Regge ([1]), obtained in quantum mechanics, can without difficulty be generalized to field theory ([2]). In the present note we wish to investigate the indicated analytic properties.

2. Let us write the integral equations of the Chew–Mandelstam type ([3]) for determining the partial amplitudes. Put

$$A_l^I(v) = D_l^I(v)^{-1} N_l^I(v).$$

Then D_l^I satisfies the equation

$$D_l^I(\omega) = 1 + \frac{1}{\pi}\int_1^\infty d\omega'\, K(\omega,\omega')\, f_l^I(\omega')\, D_l^I(\omega'), \tag{1}$$

and

$$N_l^I(v) = \frac{-1}{\pi}\int_1^\infty \frac{d\omega'}{\omega'+v}\, f_l^I(\omega')\, D_l^I(\omega'), \tag{2}$$

where

$$\omega = -v, \quad K(\omega,\omega') = \frac{2}{\omega'-\omega}\left[\left(\frac{\omega'}{\omega-1}\right)^{1/2} Q_0\left(\left(\frac{\omega'}{\omega-1}\right)^{1/2}\right) - \left(\frac{\omega}{\omega-1}\right)^{1/2} Q_0\left(\left(\frac{\omega}{\omega-1}\right)^{1/2}\right)\right],$$

$$f_l^I(v) = \frac{1}{v}\sum_{I'} \alpha_{II'} \int_0^{-v-1} A_s^{I'}\left(v',\, 1+2\frac{v'+1}{v}\right) P_l\left(1+2\frac{v'+1}{v}\right) dv', \tag{3}$$

and otherwise we follow the notation of work ([3]). The system of equations (1) –(3) is solved by iteration.

We choose the zeroth approximation for $A_s^{I'}$ so that $\lim_{v\to\infty} f_l^I(v) = 0$ (this is necessary for the self-consistency of the system ([4])). Then, as is easy to show, (1) is an equation of Fredholm type. f_l^I is, obviously, an entire function of l (since P_l is an entire function of l), and therefore D_l^I and N_l^I separately (and hence also A_l^I) are meromorphic in l. (This can be verified by writing the Fredholm solution for (1).)

3. To compute f_l^I in the second approximation, let us represent the first approximation to $\operatorname{Im} A_l^I$ in the form of a Watson–Sommerfeld integral. (This is possible in view of the results of item 2.) Then:

$$f_l^I(v) = \frac{1}{4\pi i} \sum_{I'} \alpha_{II'} \int_0^{-v-1} dv'\, P_l\left(1 + 2\frac{v'+1}{v}\right) \times$$

$$\times \left\{ \int_{-i\infty}^{i\infty} \frac{\lambda'\, d\lambda'}{\cos \lambda' \pi} \operatorname{Im} A_{\lambda'}^{I'}(v')\, P_{\lambda'-1/2}\left(-1 - 2\frac{v+1}{v'}\right) \left(1 + (-1)^{I'+1} \sin \pi\lambda'\right) - \Sigma \right\}. \tag{4}$$

Here $\lambda' = l' + 1/2$, and Σ denotes the contribution of the poles in the λ'-plane. The integral over λ' exists at least in the sense of the principal value. One can verify that the equation for D_l^I in the second approximation is of Fredholm type; therefore D_l^I in the second approximation is meromorphic in l.

4. With the aid of the procedure described above, it can be shown that $A_l^I(\nu)$ is meromorphic in l at each step of the iteration, and that the poles in the l-plane are determined by the zeros of D_l^I. It does not yet follow directly from this that, assuming convergence of the iteration, the limiting function of the iteration sequence will be meromorphic in l. It can be shown, however, that if $|A_l^I(\nu)|$ as $\nu > 0$ tends uniformly to some limiting function, then the amplitude itself is meromorphic in l.

Knowing the indicated analytic properties, we can choose the zeroth approximation in the most appropriate way, for example, by taking one Regge pole. This choice corresponds to Wong' s proposal[5].

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Note: Figure translations are in progress. See original paper for figures.