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Abstract

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PHYSICS

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ON SHOCK WAVES IN A RAREFIED PLASMA PLACED IN A WEAK MAGNETIC FIELD

(Presented by Academician A. A. Artsimovich on 3 May 1962)

At present the question of so-called "collisionless" shock waves in a plasma propagating across a strong magnetic field has been studied in detail (see ([1]); bibliography there). A magnetic field parallel to the plane of the wave front holds back the more "hot" particles, preventing the spreading of the transition region between the unperturbed ("cold") plasma (ahead of the shock-wave front) and the "heated" plasma behind the wave. For large Mach numbers the thickness of the front of such a wave is, in order of magnitude, close to the ion Larmor radius. In a number of works ([2], [3]) the possibility of the existence of collisionless shock waves also in a plasma without a magnetic field was discussed. In this case the mechanism restraining the spreading of the transition region was indicated to be the so-called "beam" instability of two interpenetrating plasmas. With such an approach, however, the thermal spread within each of the "beams" was not taken into account. A more rigorous treatment, taking thermal motion into account, does not, however, give an instability up to Mach numbers from unity to approximately $(m_i/m_e)^{1/2}$, if the "temperatures" of the electrons are comparable with the "temperature of the ions" or smaller than them*; m_i is the ion mass, m_e the electron mass (see, for example, ([4])). Below we shall show that an instability of another type (the so-called "anisotropic" one), even at small Mach numbers, must lead to the formation of a collisionless shock wave.

1. Let us first examine the question of the nature of the instability itself. To begin with, we restrict ourselves to the case when the magnetic field H_0 in the unperturbed plasma is altogether absent. We shall consider perturbations with $\operatorname{div} \mathbf{E} = 0$ (the so-called transverse ones). The initial linearized system of equations for the perturbed quantities f_1, $\mathbf{E}$, $\mathbf{H}$ has the form:

$$\frac{\partial f_{1\alpha}}{\partial t} + \mathbf{v}\frac{\partial f_{1\alpha}}{\partial \mathbf{r}} + \frac{e_\alpha}{m_\alpha}\left\{\mathbf{E} + \frac{1}{c}[\mathbf{v}\mathbf{H}]\right\}\frac{\partial f_{0\alpha}}{\partial \mathbf{v}} = 0; \tag{1}$$

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$$\text{rot}\,\mathbf{H} = \frac{4\pi}{c}\sum_\alpha e_\alpha \int \mathbf{v} f_{1\alpha}\,d\mathbf{v} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}; \tag{2}$$

$$\text{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \tag{3}$$

where $f_{0\alpha}$ is the unperturbed distribution function ($\alpha = i, e$). As usual, we choose the perturbation in the form $\sim \exp i(\mathbf{kr} - \omega t)$. In general, $f_{0i}(\mathbf{v})$ is some anisotropic function. However, for definiteness we choose it in the form

$$f_{0i} = \left(\frac{m_i}{2\pi T_\perp}\right)\left(\frac{m_i}{2\pi T_\parallel}\right)^{1/2} \exp\left\{-\frac{m_i v_\perp^2}{2T_\perp} - \frac{m_i v_\parallel^2}{2T_\parallel}\right\}. \tag{4}$$

* The case of electrons very "hot" in comparison with the ions ($T_e \gg T_i$), as shown in ([5]), leads to a peculiar "collisionless" shock wave, but for an entirely different reason.

The results obtained will also be qualitatively valid for a more general distribution function $f_0(v_\perp, v_\parallel)$, if $T_\perp$ and $T_\parallel$ are understood as

$$\frac{m_i}{2}\overline{(v_\perp - v_\perp)^2} \quad \text{and} \quad \frac{m_i}{2}\overline{(v_\parallel - v_\parallel)^2}.$$

We shall take the electron distribution to be isotropic (this will be justified below) and, for simplicity, Maxwellian. Choosing the y-axis along $\mathbf{H}$, the z-axis along $\mathbf{k}$, and substituting the solution (1) into (2), after standard calculations we obtain the following dispersion equation (for $|\omega| \ll kc$):

$$k_0^2 - k^2 = -i\sqrt{\frac{\pi}{2}}\frac{\omega_{0i}^2}{c^2}\sqrt{m_i}\left[\frac{T_\perp}{T_\parallel}\frac{1}{\sqrt{T_\parallel}} + \frac{1}{\sqrt{T_e}}\sqrt{\frac{m_i}{m_e}}\right]\frac{\omega}{k}. \tag{5}$$

Here

$$\omega_{0i}^2 = \frac{4\pi e^2 n}{m_i}, \qquad k = k_z, \qquad k_0^2 = \frac{\omega_{0i}^2}{c^2}\frac{\Delta T}{T_\parallel} \tag{6}$$

($\Delta T = T_\perp - T_\parallel > 0$; n is the particle density).

It follows from (5) that growing solutions arise for $k < k_0$, and the instability is aperiodic in character. In the case when $T_\perp / T_\parallel \ll \sqrt{m_i/m_e}$, the minimum growth time is

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$$\tau_{\alpha\,\min} \sim \left(\frac{T_\parallel}{\Delta T}\right)^{3/2} \frac{c}{\omega_{0i}} \sqrt{\frac{m_i}{T_e}} \sqrt{\frac{m_i}{m_e}}, \tag{7}$$

and it is easy to see that neglect of collisions is legitimate in practically interesting cases. For "oblique" waves ($k_z, k_x \neq 0$), the curve giving the stability boundary has the form

$$\Delta T \frac{\omega_{0i}^2}{c^2} \left(k_z^2 T_\parallel - k_x^2 T_\perp\right) = \left(k_x^2 T_\perp + k_z^2 T_\parallel\right)^2, \tag{8}$$

and the instability arises in the interior region of the curve (see Fig. 1), where the increments are, in order of magnitude, the same as those calculated from (5).

Fig. 1. $\tan\varphi = \sqrt{T_\perp / T_\parallel}$

It can be shown that the instability under consideration can be obtained in the hydrodynamic approximation for multivelocity flows (which is possible in a rarefied plasma). Namely, if four groups of particles are considered, each of which moves in a definite direction along one of the axes (x or z) with an average speed of the order of the thermal speed and has different temperatures along the axes, then an instability develops with the boundary k_0 indicated in (6). If, in this case, the corresponding "Ohm' s law" for the electrons is used,

$$\mathbf{j} = \sigma \left\{\mathbf{E} + \frac{1}{c}[\mathbf{v}_i \mathbf{H}]\right\}, \tag{9}$$

where

$$\sigma \sim \frac{e^2 n}{m_e^{1/2} k_0 T_e^{1/2}}, \qquad (\mathbf{j} = en\overline{\mathbf{v}}_e), \tag{10}$$

then the correct value is also obtained for the increment ω (the ion current created by the electric field may be neglected if $\Delta T \sim T_\parallel$). We note the analogy of (10) with the formula for the conductivity in the anomalous skin effect.

Fig. 2. O, a are points where $H = 0$; the arrows indicate the directions of particle motion near point a.

The mechanism of this instability is easy to understand from the following model. Consider two groups of particles moving in opposite directions along the x-axis (see Fig. 2, top view). Then, owing to the redistribution of the motion of the particles in the inhomogeneous perturbation field $\mathbf{H}$, an electric current arises, directed opposite to the electric field; this leads to growth of the perturbation if the current produced by the electric field itself is smaller.

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2. Passing to the question of the possibility of formation of a shock wave owing to an aperiodic instability, let us consider the following example. Suppose that a perturbation has arisen in some region of a rarefied plasma. In the absence of any restraining mechanism, with time the perturbation would have spread out because of the gradual escape of the faster particles. In fact, when particles enter new regions, an anisotropy of the velocity distribution arises there, and with it the associated instability; the growing magnetic field retains the particles, as is readily seen, until the energies are equalized. Thus there is created the possibility for propagation of a nonspreading perturbation which, for a sufficiently large anisotropy of temperatures ($\Delta T \gg T_{\|}$), has the character of a shock wave. To clarify the features of the shock wave of interest to us, let us estimate the amplitude of the pulsations H_a in the nonlinear regime of the developing instability, when

$$k_0 v f_{1e} \sim \frac{e}{m_e c}\,[\mathbf{v}\mathbf{H}_a]\,\frac{\partial f_{1e}}{\partial \mathbf{v}}. \tag{11}$$

On the right-hand side of (11) stands the nonlinear term which, as is easy to see, leads to the generation of new perturbation waves, and for the growth of which the waves with the smallest amplification time are mainly responsible ($\tau_{a\,\min}$ is attained at $k = 0.6\,k_0$). From (11) we have for the amplitude of the pulsations

$$H_a \sim \sqrt{(m_e/m_i)}\,\sqrt{nT_e}\,\sqrt{\Delta T/T_{\|}}. \tag{12}$$

Let us note that (12) corresponds to those magnetic fields for which

$$r_e \sim 1/k_0 \tag{13}$$

(r_e is the Larmor radius of the electrons). Hence the physical reason for the impossibility of fields substantially larger than (12) is clear: in them the drift approximation would be applicable to the electrons, which would lead to excessive heating of the electrons (the energy of the electrons could exceed the energy stored in the ions).

As a result of the back reaction of the perturbation, the initial distribution f_{0i} changes with time. As is seen from (13), a moving ion "collides" with small-scale pulsations and therefore is only weakly deflected; for f_{0i}, consequently, the Fokker–Planck equation is valid. The terms responsible for the isotropization of the initial distribution, in order of magnitude, give

$$\frac{\partial f_{0i}}{\partial t} \sim D\,\frac{\partial^2 f_{0i}}{\partial v^2}, \tag{14}$$

where

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$$D \sim e^2 H_a^2 / m_i^2 c^2 k_0^2 \tau_H, \tag{15}$$

and τ_H is the transit time ($\sim 1/kv$). The coefficient D in the equation for f_{0i} corresponds to the term

$$St_H = \frac{2e^2}{m_i^2 c^2} \sum_k |H^2| \, \delta(\mathbf{kv}) \left(v_x \frac{\partial}{\partial v_z} - v_z \frac{\partial}{\partial v_x} \right) \left(v_x \frac{\partial f_{0i}}{\partial v_z} - v_z \frac{\partial f_{0i}}{\partial v_x} \right), \tag{16}$$

obtained from the kinetic equation by averaging over the small-scale pulsations of the terms connected only with the magnetic field of the perturbation.

Estimating, with the aid of (15), T—the time of isotropization of the ions due to "collisions" with field inhomogeneities in the nonlinear regime ($D \sim v^2/T$)—we obtain, as a result, for the width of the shock wave Δ_0:

$$\Delta_0 \sim v_i T \sim \left(\frac{m_i}{m_e} \right) \frac{c}{\omega_{0i}} \left(\frac{T_\parallel}{\Delta T} \right)^{1/2} \sqrt{\frac{T_\parallel}{T_e}} \left(\frac{T_\perp}{T_e} \right)^{1/2}. \tag{17}$$

As is not difficult to show, the isotropization time of the electrons proves to be $(m_e/m_i)^2$ times smaller than for the ions, which also justifies the choice of an isotropic distribution function for the electrons.

It is important to note that the birth of new scales in the nonlinear regime in the second approximation is possible only for oblique waves: this is evident, for example, from formula (9)—the nonlinear term contributes along the x-axis if there is a velocity along the z-axis, and conversely. Therefore the turbulence that arises is essentially two-dimensional in character, which complicates the solution of the integral equation for the perturbation fields.

Fig. 3.

$$H_a' = \left(\frac{m_e}{m_i} \right) \sqrt{n T_e} \sqrt{\frac{\Delta T}{T_\parallel}}.$$

An analogous consideration in the presence of initial frozen-in fields H_0, satisfying the condition

$$\frac{m_e}{m_i} \sqrt{n T_e} \sqrt{\frac{\Delta T}{T_\parallel}} \ll H_0 \ll \sqrt{n T_\parallel}, \tag{18}$$

shows that the shock-wave width Δ must be of the order of the ion Larmor radius in the magnetic field of the perturbation, whose amplitude turns out to be of the same order as H_0 (see Fig. 3). This difference in the results is due

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to the fact that even a weak initial frozen-in field changes the increment and the spatial boundary of stability (since the character of the electron motion changes).

It is known (see, for example, [6]) that magnetic storms are caused by streams of particles coming from the Sun with a sharp leading front of thickness 100–200 thousand km, having the character of a shock wave in the interplanetary gas. For ion velocities in the stream of 10^8 cm/sec and an interplanetary-gas density of 10^2 cm^{-3}, their mean free path due to collisions is 10^{13} km. This shows that the given shock wave can arise only due to self-consistent fields in the ionized interplanetary gas.

It is interesting to note that, even under the assumption of the absence of frozen-in fields, from (17) we obtain a reasonable upper bound for the thickness of the shock wave at the density of the interplanetary gas and $\Delta T \gtrsim T_\parallel,\ T_e \sim T_\parallel$.

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Note: Figure translations are in progress. See original paper for figures.

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