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Abstract

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The Influence of Pair Correlations of the Superconducting Type on the Rates of α-Decays

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As is known ([1–3]), pair correlations of nucleons of the superconducting type exert a very strong influence on the properties of the ground and excited states of atomic nuclei. Therefore they play a major role in the processes of β- and γ-transitions in nuclei. There is no doubt that pair correlations must have a noticeable influence on the rates of α-decays. Detailed theoretical investigations of the probabilities of α-transitions have been carried out in many works, for example in ([4–6]); however, in them the influence of the superfluidity of the ground and excited states on α-decay was not taken into account. In the present work we formulate a theory of α-decay within the framework of the superfluid model of the nucleus and investigate the influence of pair correlations of the superconducting type both on the absolute probabilities of α-decays and, in particular, on the hindrance factors F.

The matrix element of α-decay of the parent nucleus with wave function $\Psi = \Psi(N)\cdot\Psi(z)$, taken in the form of a product of the wave functions of the proton and neutron systems, into the daughter nucleus with $\Psi = \Psi(N-2)\Psi(z-2)$, we write in the form

$$M = \Psi^*(N-2)\Psi(z-2)A\Psi(z)\Psi(N). \tag{1}$$

The operator A, describing the emission of an α-particle, we represent as follows:

$$A = \frac{1}{4}\sum_{\substack{\nu,\nu';\omega,\omega'\\ \tau,\tau';\sigma,\sigma'}} W_{\tau,\tau';\sigma,\sigma'}(p\nu,\nu'|n\omega,\omega')\, a_{\nu\tau}a_{\nu'\tau'}b_{\omega\sigma}b_{\omega'\sigma'}, \tag{2}$$

where $a_{\nu\tau}, b_{\omega\sigma}$ are the annihilation operators of a proton and a neutron, $\tau = \pm1$, $\sigma = \pm1$, with states differing by the sign of τ or σ being conjugate with respect to the time-reversal operation; the summation over ν,ν' (ω,ω') is carried out over the one-particle proton (neutron) levels of the mean field. The function W describes both the passage of the α-particle through the potential barrier and the probability of its formation.

Let us find the matrix element of α-decay of an even-even nucleus between ground states. Using the wave functions ([2])

$$\Psi = \prod_s (u_s + v_s a^+_{s+} a^+_{s-})\Psi_0,$$

which take into account pair correlations of nucleons, we obtain

$$M = \sum_{\nu,\omega} W_{+-;+-}(p\nu,\nu|n\omega,\omega)\, u_\nu(z-2)v_\nu(z) \prod_{s\neq\nu}(u_s(z-2)u_s(z) + v_s(z-2)v_s(z)) \cdot u_\omega(N-2)v_\omega(N) \prod_{s'\neq\omega}(u_{s'}(N-2)u_{s'}(N) + v_{s'}(N-2)v_{s'}(N)), \tag{3}$$

where

$$u_s(z)^2 = \frac{1}{2}\left\{1 + \frac{E(s)-\lambda(z)}{[C(z)^2 + \{E(s)-\lambda(z)\}^2]^{1/2}}\right\}; \quad v_s(z)^2 = 1 - u_s(z)^2;$$

$C(z)$ is the correlation function; $\lambda(z)$ is the chemical potential for the ground state of a system of z protons; $E(s)$ is a one-

single-particle energy levels. In the case where pair correlations are absent, (3) takes the form

$$M = W_{+-;+-}(p\nu = K(z), \nu = K(z) \mid n\omega = K(N), \omega = K(N)), \tag{4}$$

where $K(z)$ denotes the last occupied level of the system of z protons in the absence of pair correlations, $K-1$ the first hole level, $K+1$ the first particle level, etc. From (4) it is clear that, if pair correlations are absent, then the α-particle can be formed only from two neutrons and two protons, and each is located on the last occupied level of the mean field. Since the probability of formation of an α-particle in a nucleus is proportional to the overlap integral of the corresponding wave functions, it should change substantially in going from one nucleus to another because of the change in the quantum numbers of the level K, which is not observed experimentally. The effect of pair correlations leads to the fact that α-particles can be formed with appreciable probability from pairs located in many states both below and above the level K, i.e., that α-decay contains an averaged contribution from many levels near the surface of the Fermi sphere. This leads, first, to an increase in the probability of α-decay and, second, to a decrease in the change in the probability of α-particle formation in the transition from nucleus to nucleus.

To separate the effects associated with pair correlations of nucleons in nuclei from the influence of other factors in α-decays, let us consider the approximation: the diagonal part of the function W, both with respect to the quantum numbers of protons and with respect to the quantum numbers of neutrons, does not depend on these quantum numbers, i.e.

$$W_{+-;+-}(p\nu,\nu \mid n\omega,\omega) = W(p \mid n), \tag{5}$$

$$W_{+-;\sigma_1,\sigma_2}(p\nu,\nu \mid n\omega_1,\omega_2) = W_{\sigma_1\sigma_2}(p \mid n\omega_1,\omega_2), \tag{5'}$$

Apparently, in estimating the influence of pair correlations on α-decay such an averaged treatment is correct.

The matrix element (3) in the approximation (5) takes the form:

$$M = W(p \mid n) R_N^{1/2} R_z^{1/2}, \tag{6}$$

$$R_z^{1/2} = \sum_\nu u_\nu(z-2)v_\nu(z) \prod_{s\neq\nu} \left(u_s(z-2)u_s(z) + v_s(z-2)v_s(z)\right). \tag{7}$$

If one uses the values of C and λ obtained in (7), then, for example, for the α-decay of Cm^{244} to the ground state of Pu^{240} we obtain $R_N = 38$, $R_z = 45$, and $R_N R_z = 1700$. Calculations show that for nuclei in the region $230 \leq A \leq 254$ the values of $R_N R_z$ lie within the limits $1500 < R_N R_z < 3000$. Let us note that, possibly, in α-decays of nuclei located near closed shells, the reduction of the reduced transition probabilities occurs partly because of the disappearance of pair correlations in the system with a closed shell.

Let us find the matrix element of an α-transition to two-quasiparticle excited states of an even-even nucleus. For α-decay to a neutron state with quasiparticles on the levels f_1 and f_2 ($f_1 \neq f_2$) we obtain:

$$M(f_1,f_2) = W_{\sigma_1\sigma_2}(p \mid nf_1,f_2) R_z^{1/2} R_N(f_1,f_2)^{1/2}, \tag{8}$$

$$R_N(f_1,f_2) = v_{f_1}(N)^2 v_{f_2}(N)^2 \prod_{s\neq f_1,f_2} (u_s(N-2,f_1,f_2)u_s(N) + \\ + v_s(N-2,f_1,f_2)v_s(N))^2, \tag{9}$$

where $R_N(f_1,f_2) < 1$. According to the superfluid model of the nucleus, in this case the α-particle is formed only from neutrons located in the states f_1 and f_2, and the α-decay rate is proportional to the density of neutrons $v_{f_1}^2, v_{f_2}^2$ in these states of the parent nucleus. The hindrance factor $F = (M/M(f_1, f_2))^2$ is obtained in the form

$$F = \left(W(p \mid n)/W_{\sigma_1\sigma_2}(p \mid nf_1, f_2)\right)^2 R_N/R_N(f_1, f_2). \quad (10)$$

Thus, from the superconducting model of the nucleus it follows that the reduced probabilities of α-transitions to two-quasiparticle states of even-even nuclei are decreased by a factor $R_N/R_N(f_1, f_2)$ in comparison with α-decay to the ground state. Thus, for the α-decay of Cm^{244} to two-quasiparticle states of Pu^{240} with an energy of 2 MeV, the ratio $R_N/R_N(f_1, f_2)$ takes values in the interval 150–500. We note that in this case the blocking effect plays a noticeable role.

Let us consider favored α-decay of odd nuclei, in which the quasiparticle is on one and the same level in the parent and daughter nuclei. The decay matrix element, when the odd neutron is on the level f, will be written as

$$M(f) = W(p \mid n) R_2^{1/2} R_{N+1}(f)^{1/2}, \quad (11)$$

$$R_{N+1}(f)^{1/2} = \sum_{\omega \neq f} u_\omega(N-1, f) v_\omega(N+1, f) \prod_{s \neq \omega, f} \left(u_s(N-1, f) u_s(N+1, f) + v_s(N-1, f) v_s(N+1, f)\right). \quad (12)$$

The hindrance factor is obtained in the form

$$F = \left[M(N)^2 + M(N+2)^2\right]/2M(N+1, f)^2 = \left[R_N + R_{N+2}\right]/2R_{N+1}(f). \quad (13)$$

The hindrance factor for favored α-decays, associated with the blocking effect, in the transuranium region varies within the limits

$$1.2 < \left[R_N + R_{N+2}\right]/2R_{N+1}(f) < 3, \quad (14)$$

and $F > 3$, as a rule, cannot be explained in this way. In Table 1 we compare the calculated values of F with experimental data; the values of F placed in parentheses were calculated on the basis of the data in (11), taking into account that, in addition to α-particles emitted with $l = 0$, some of them are emitted with $l = 2$ and $l = 4$. From Table 1 it is seen that the calculations are in satisfactory agreement with the experimental data.

Table 1

Favored α-decays

State	α-decay	F, experiment	F, calculation	State	α-decay	F, experiment	F, calculation
$9/2-$ [734]	$^{151}Cf^{249} \rightarrow$ $^{149}Cm^{245}$	$1.8^{(8)}$	1.8	$3/2-$ [521]	$^{97}Bk^{245} \rightarrow$ $^{95}Am^{241}$	$1.7^{(8)}$	1.7
$7/2-$ [624]	$^{149}Cm^{245} \rightarrow$ $^{147}Pu^{241}$	$2.2^{(8)}$	2.0	$5/2-$ [523]	$^{95}Am^{243} \rightarrow$ $^{93}Np^{239}$	$1.1^{(8)}$	1.7
$5/2+$ [622]	$^{147}Cm^{243} \rightarrow$ $^{145}Pu^{239}$	$\begin{cases}1.5^{(8)}\\ 2^{(9)}\end{cases}$	$\begin{cases}2.2\\ (1.8)\end{cases}$	$5/2-$ [523]	$^{95}Am^{241} \rightarrow$ $^{93}Np^{237}$	$\begin{cases}1.3^{(8)}\\ 2^{(9)}\end{cases}$	$\begin{cases}1.7\\ (1.4)\end{cases}$
$1/2+$ [631]	$^{145}Cm^{241} \rightarrow$ $^{143}Pu^{237}$	$2.7^{(8)}$	2.1	$5/2-$ [523]	$^{95}Am^{239} \rightarrow$ $^{93}Np^{235}$	$2.3^{(8)}$	1.7
$1/2+$ [631]	$^{145}Pu^{239} \rightarrow$ $^{143}U^{235}$	$\begin{cases}2.5^{(8)}\\ 1.7^{(10)}\end{cases}$	2.1	$5/2+$ [642]	$^{93}Np^{237} \rightarrow$ $^{91}Pa^{233}$	$\begin{cases}3.8^{(8)}\\ 5^{(13)}\end{cases}$	1.4

Let us consider unfavored α-decays, when the quasiparticle passes from one state to another. The matrix element of α-decay, when the neutron passes from the state f_2 to the state f_1 ($f_2 \neq f_1$), is obtained in the form

$$M(f_1, f_2) = W_{\sigma_1,-\sigma_2}(p \mid nf_1, f_2) R_2^{1/2} R_{N+1}(f_1, f_2)^{1/2}, \tag{15}$$

where

$$R_{N+1}(f_1, f_2) = u_{f_2}(N-1, f_1)^2 v_{f_1}(N+1, f_2)^2 \prod_{s \neq f_1, f_2} \left(u_s(N-1, f_1) u_s(N+1, f_2) + v_s(N-1, f_1) v_s(N+1, f_2)\right)^2.$$

In the case of an unfavored α-decay of an odd nucleus, the α-particle is formed from a number of proton pairs lying at levels near the Fermi surface, and from neutrons in the states f_1 and f_2. Therefore unfavored α-decays are strongly retarded in comparison with favored ones. The hindrance factor is found in the form

$$F = \left(W(p \mid n)/W_{\sigma_1,-\sigma_2}(p \mid nf_1, f_2)\right)^2 \left[R_N + R_{N+2}\right]/2R_{N+1}(f_1, f_2). \tag{16}$$

The ratio $\left[R_N + R_{N+2}\right]/2R_{N+1}(f_1, f_2) = \eta$ for α-transitions to the ground and rotational states of a strongly deformed nucleus takes values in the range 50–130; for α-decays to a particle $(K+2)$-state it is equal to 200–800, and in $(K+3)$ and higher states the ratio exceeds

10^3. Unfavorable α decays to particle excited states are more strongly retarded than transitions to hole states.

The role of pair correlations in (16) is demonstrated in Table 2. It is seen from the table that pair correlations are responsible only for part of the decrease in the reduced transition probabilities; the remaining part $\{W(p \mid n)/W_{\sigma_1,-\sigma_2}(p \mid nf_1, f_2)\}^2$ is associated both with the decrease in the probability of formation of the α particle from nucleons located in different orbits and with a number of other phenomena. Pair correlations make a significant contribution to F; therefore, instead of systematizing the quantities F as functions of the quantum numbers of the states f_1 and f_2, as in (12), one should carry out such a systematization of the quantities $\{W(p \mid n)/W_{\sigma_1,-\sigma_2}(p \mid nf_1 f_2)\}^2$.

Table 2

Unfavorable α decays

f_2	α decay	f_1	F, experiment	η
$9/2-$ [734] K	$Cf^{249} \to Cm^{245}$	$7/2+$ [624] K	833 (8)	80
$9/2-$ [734] K	$Cf^{249} \to Cm^{245}$	$5/2+$ [622] $K-1$	251 (8)	80
$5/2+$ [622] K	$Cm^{243} \to Pu^{239}$	$1/2+$ [631] K	3220 (8)	50
$7/2-$ [743] K	$Pu^{237} \to U^{233}$	$7/2-$ [743] $K-1$	143 (8)	60
$7/2-$ [743] K	$Pu^{237} \to U^{233}$	$5/2+$ [633] K	1059 (8)	70
$3/2-$ [521] K	$Bk^{245} \to Am^{241}$	$5/2-$ [523] K	342 (8)	60
$3/2-$ [521] K	$Bk^{245} \to Am^{241}$	$5/2+$ [642] $K-1$	29.4 (8)	57
$3/2-$ [521] K	$Bk^{243} \to Am^{239}$	$5/2-$ [523] K	682 (8)	60
$3/2-$ [521] K	$Bk^{243} \to Am^{239}$	$5/2+$ [642] $K-1$	68 (8)	57
$5/2-$ [523] K	$Am^{243} \to Np^{239}$	$5/2+$ [642] K	1570 (8)	120
$5/2-$ [523] K	$Am^{243} \to Np^{239}$	$3/2-$ [521] $K+2$		760
$5/2-$ [523] K	$Am^{243} \to Np^{239}$	$1/2-$ [530] $K-1$	1526 (8)	120
$5/2+$ [642] K	$Np^{237} \to Pa^{233}$	$1/2-$ [530] K	3400 (13)	115

Unfavorable α decays give information on the nondiagonal parts of the function

W, which can be used to calculate hindrance coefficients in α decays both of odd-odd nuclei and into two-quasiparticle states of even-even nuclei.

Thus, pair correlations of nucleons of the superconducting type have a very large influence both on the absolute probabilities of α decays to the ground states of even-even nuclei and of favored decays in odd nuclei, and especially on the values of the hindrance coefficients in unfavorable α decays and in α transitions to two-quasiparticle levels of even-even nuclei. Taking pair correlations into account leads to improved agreement between theory and the corresponding experimental data.

The results obtained are direct consequences of those properties of the ground and excited states that follow from the superconducting model of the nucleus. The numerical calculations performed use characteristics of the properties of the ground and excited states obtained in (7), and are completely unambiguous, since they contain not a single new parameter. We note that the proposed method can be easily generalized to the case of α decays to collective levels 0+, 2+, 0−.

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Note added in proof. After the article had been submitted for publication, I became aware of a preprint by Mang and Rasmussen in which the influence of pair correlations on the probabilities of α transitions to rotational bands of the ground states of even-even nuclei is taken into account.

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REFERENCES

1. V. G. Solov' ev, DAN, **133**, 325 (1960); JETP, **40**, 654 (1961).

2. V. G. Solov' ev, Izv. AN SSSR, ser. fiz., **25**, 1198 (1961).

3. V. G. Solov' ev, Mat. Fys. Skr. Dan. Vid. Selsk., **1**, No. 11 (1961); V. G. Solov' ev, Dissertation, Preprint of the Joint Institute for Nuclear Research, R-801, 1961.

4. A. Bohr, P. O. Froman, B. R. Mottelson, Dan. Mat. Fys. Medd., **29**, 10 (1955).

5. I. Perlman, J. O. Rasmussen, *Alpha Radioactivity*, IL, 1959.

6. V. G. Nosov, JETP, **39**, 141 (1960).

7. T. Vereshch, V. G. Solov' ev, T. Shiklosh, Izv. AN SSSR, ser. fiz., in press.

8. J. O. Rasmussen, Phys. Rev., **115**, 1675 (1959).

9. V. Mottelson, S. G. Nilsson, Mat. Fys. Skr. Dan. Vid. Selsk., **1**, No. 8 (1959).

10. B. S. Dzhelepov, R. B. Ivanov, V. G. Nedovesov, JETP, **41**, 1725 (1961).

11. A. Sandulescu, M. Iosifescu, Nucl. Phys., **26**, 209 (1961).

12. O. Prior, Ark. f. Fys., **16**, 15 (1959).

13. S. A. Baranov, V. M. Kulakov et al., JETP, **41**, 1733 (1961).

Note: Figure translations are in progress. See original paper for figures.