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THEORY OF ELASTICITY

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Abstract

Full Text

THEORY OF ELASTICITY

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QUALITATIVE STUDY OF A ROUND SYMMETRICALLY COMPRESSED PLATE UNDER A LARGE EDGE LOAD

(PROOF OF THE APPEARANCE OF A CORRUGATION)

(Presented by Academician V. I. Smirnov on 21 V 1962)

The possibility that a circular plate, under certain symmetric loads, may develop an asymmetric deflection (a corrugation) has long been known experimentally. Theoretically, the question of the appearance of a corrugation in a symmetrically loaded circular plate was considered in work ⁽¹⁾: the authors investigated the Kármán equation for a circular rigidly clamped plate in the absence of tangential forces on the contour and under a constant transverse load. However, the existence of an asymmetric solution cannot be regarded as rigorously proved, since the differential equation of equilibrium was satisfied only approximately.

In the present paper we shall consider a circular plate, rigidly supported, uniformly compressed along the contour, in the absence of transverse loading.

Mathematically, the state of the plate is described by the system of Kirchhoff equations

$$D\Delta^2 w - h\sigma_r w_{rr} - h\sigma_\theta \left(\frac{1}{r} w_r + \frac{1}{r^2} w_{\theta\theta} \right) - 2h\tau_{r\theta} \left(-\frac{w_\theta}{r^2} + \frac{w_{r\theta}}{r} \right) = 0; \quad (1a)$$

$$\begin{aligned} 2u_{rr} + \frac{2u_r}{r} - \frac{2u}{r^2} + \frac{1-\sigma}{r^2} u_{\theta\theta} + \frac{1+\sigma}{r} v_{r\theta} + \frac{3\sigma-1}{r^2} v_\theta + 2w_r w_{rr} \\ + \frac{1-\sigma}{r} w_r^2 + \frac{1+\sigma}{r^2} w_\theta w_{r\theta} - \frac{1+\sigma}{r^3} w_\theta^2 + \frac{1-\sigma}{r^2} w_r w_{\theta\theta} = 0; \end{aligned} \quad (1b)$$

$$\begin{aligned} (1-\sigma) \left(v_{rr} + \frac{v_r}{r} + \frac{v}{r^2} \right) - \frac{2v_{\theta\theta}}{r^2} - \frac{1+\sigma}{r} u_{r\theta} - \frac{3-\sigma}{r^2} u_\theta - \frac{2w_\theta w_r}{r^2} \\ - \frac{2w_\theta w_{\theta\theta}}{r^3} + \frac{1+\sigma}{r^2} w_r w_\theta - \frac{1+\sigma}{r} w_r w_{r\theta} - \frac{1-\sigma}{r} w_\theta w_{rr} = 0 \end{aligned} \quad (1c)$$

with boundary conditions

$$w|_{r=R} = 0; \quad (2a)$$

$$\Delta w - \frac{1-\sigma}{R} w_r - \frac{1-\sigma}{R^2} w_{\theta\theta} \Big|_{r=R} = 0; \quad (2b)$$

$$\sigma_r|_{r=R} = -p; \quad (2c)$$

$$\tau_{r\theta}|_{r=R} = 0, \quad (2d)$$

where w is the deflection; u is the radial displacement; v is the tangential displacement; $\sigma_r, \sigma_\theta, \tau_{r\theta}$ are stresses; θ is the polar angle.

Consider the functional

$$I = \frac{Eh^3}{24(1-\sigma^2)} \iint_{r \leq R} \left\{ (\Delta w)^2 - 2(1-\sigma) \left[\frac{1}{r} w_r w_{rr} + \frac{1}{r^2} w_{rr} w_{\theta\theta} - \left(\frac{w_{r\theta}}{r} - \frac{w_\theta}{r^2} \right)^2 \right] \right\} r dr d\theta + \frac{Eh}{2(1-\sigma^2)} \iint_{r \leq R} \left[(\varepsilon_{rr} + \varepsilon_{\theta\theta})^2 - 2(1-\sigma) \left(\varepsilon_{rr} \varepsilon_{\theta\theta} - \frac{\varepsilon_{r\theta}^2}{4} \right) \right] r dr d\theta, \quad (3)$$

where $\varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{r\theta}$ are determined through u, v, w by the well-known Kirchhoff formulas. Here v and w are arbitrary sufficiently smooth functions ($w(R, \theta) = 0$), while u is determined through v and w from equation (1) with the boundary condition (2).

Solving problem (1), (1), (1), (2), (2), (2), (2) is equivalent to finding the stationary points of the functional $I(v, w)$, since equations (1) and (1) are the Euler equations for $I(v, w)$, and conditions (2) and (2) are natural.

It is easy to prove that the functional (3) is bounded below; then it can be proved, analogously to (2), that there exist v and w realizing the minimum of the functional.

In paper (3) the existence of a triple w_0, u_0, v_0 ($v_0 \equiv 0$), realizing the minimum of the functional (3) among symmetric functions, was proved.

The existence of a nonsymmetric solution will be proved if it is possible to choose such a triple of sufficiently smooth functions

$$w(r, \theta) = w_0(r) + w'(r, \theta), \quad u(r, \theta) = u_0(r) + u'(r, \theta), \quad v(r, \theta) = v'(r, \theta)$$

($w(R, \theta) = 0$, $u(r, \theta)$ are determined through w and v), which gives I a value smaller than I_0 , where $I_0 = I(u_0, v_0, w_0)$.

We shall use the Friedrichs–Stoker asymptotics ⁽³⁾. On the interval

$$\left[\frac{R}{\sqrt{1 + 2\beta_0/\lambda}}, R \right]$$

for large λ the following formulas hold:

$$\begin{aligned} \frac{dw_0}{dr} &= -\lambda^2[1.61\eta + \delta_1], & \frac{d^2w_0}{dr^2} &= -\frac{\lambda^2\eta \cdot 1.61}{R} + \frac{\lambda^3\eta\delta_2}{R}, \\ \frac{u_0}{r} &= \lambda^3[1.61\eta^2 + \delta_3], & \frac{du_0}{dr} &= -\lambda^4 \left[\frac{\eta^2(1.61)^2}{2} + \delta_4 \right], \end{aligned} \quad (4)$$

where

$$\eta = \frac{h}{R} \sqrt{12(1 - \sigma^2)}, \quad \lambda^2 = \frac{p}{\eta^2};$$

β_0 is a certain fixed number; $\delta_1, \delta_2, \delta_3, \delta_4$ can be made smaller than any $\varepsilon > 0$ for sufficiently small β_0 .

We choose the function $w'(r, \theta)$ in the following way:

$$w'(r, \theta) = a(r) \sin n\theta; \quad a(r) = \begin{cases} 0, & \text{for } r \in [0, r_0], \\ (R - r^2)(r - r_0)^4, & \text{for } r \in [r_0, R], \end{cases}$$

where

$$r_0 = \frac{R}{\sqrt{1 + 2\beta_0/\lambda}}.$$

The function $w'(r, \theta)$ is four times continuously differentiable and satisfies the condition $w'(R, \theta) = 0$. We choose $u'(r, \theta)$ and $v'(r, \theta)$ so that equation (1) and the boundary conditions (1) are satisfied. This can be done for $0 \leq \sigma < 1/3$. We substitute into

$$\Delta I = I(w(r, \theta), u(r, \theta), v(r, \theta)) - I(w_0(r), u_0(r), v_0(r))$$

the triple thus found. After integration we obtain a polynomial in n and λ , with the coefficient of the leading term being less than zero. Consequently, for sufficiently large n and λ , $\Delta I < 0$, as was required to prove.

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Note: Figure translations are in progress. See original paper for figures.

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