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THEORY OF ELASTICITY

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Abstract

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THEORY OF ELASTICITY

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A METHOD FOR SOLVING STATICALLY INDETERMINATE PROBLEMS OF PLANE FLOW OF IDEALLY PLASTIC BODIES

(Presented by Academician Yu. N. Rabotnov on 23 X 1961)

In statically indeterminate problems of plane flow of ideally plastic bodies, the number of boundary conditions for the velocities is excessive. Since the distribution of velocities on the plane of characteristics can be determined before the stress field is found, it is possible to obtain equations determining the missing boundary conditions for the stresses. The method is applicable in cases where the number and arrangement of the plastic regions are known. This method was used by the author to solve the problem of indentation of an ideally plastic strip by a punch with a convex base (^{1}).

Let us consider, as an example, the problem of drawing a sheet through a die with smooth curvilinear walls (Fig. 1). Along the contact line A_0A_* we have the relation

$$\theta - \varphi = \frac{1}{4}\pi; \tag{1}$$

here

$$\varphi = \frac{1}{2}\operatorname{Arc\,tg}\frac{\sigma_y - \sigma_x}{2\tau_{yx}},$$

θ is the angle between the Ox axis and the tangent to the line A_0A_*, measured counterclockwise.

Following Hill (^{2}), introduce the quantities

$$\alpha = \frac{1}{2}\left[\frac{p_0}{2k} - \left(\frac{p}{2k} - \varphi\right)\right], \qquad \beta = \frac{1}{2}\left[-\frac{p_0}{2k} + \left(\frac{p}{2k} + \varphi\right)\right]; \tag{2}$$

here $p = -(\sigma_x + \sigma_y)/2$.

Along the characteristics of the first and second families, the relations $p + 2k\varphi =$ const and $p - 2k\varphi =$ const, respectively, hold (^{2}).

If the characteristics in all plastic regions are curvilinear, then the projections of the velocity v_α, v_β onto the directions α, β are single-valued functions of the quantities α, β and satisfy the equation $(\partial^2 f/\partial\alpha\,\partial\beta) + f = 0$ and the boundary conditions:

for $\beta = 0$ (on L_0O, see Fig. 1)

$$\frac{\partial v_\alpha}{\partial\alpha} = (1-\varepsilon)\cos\left(\frac{1}{4}\pi + \alpha\right), \qquad \frac{\partial v_\beta}{\partial\alpha} = -(1-\varepsilon)\sin\left(\frac{1}{4}\pi + \alpha\right),$$

$$\varepsilon = \frac{H_0 - H_1}{H_0};$$

for $\alpha = 0$ (on OL_*)

$$\frac{\partial v_\alpha}{\partial\beta} = \cos\left(\frac{1}{4}\pi + \beta\right), \qquad \frac{\partial v_\beta}{\partial\beta} = -\sin\left(\frac{1}{4}\pi + \beta\right);$$

for $\alpha = 0,\ \beta = 0$ (at point O)

$$v_\alpha = 1/\sqrt{2}, \qquad v_\beta = (1-\varepsilon)/\sqrt{2}.$$

Applying Riemann’ s formula, we determine v_α and v_β in the entire region of variation of the parameters α, β. On the line A_0A_* the normal component of the velocity is zero. Satisfying this condition, after certain transforma-

we obtain the equation

$$\cos\left(\theta + \frac{1}{4}\pi\right) + \varepsilon\left\{\int_0^\alpha J_0[2\sqrt{(\alpha-\lambda)\beta}]\cos\lambda\,d\lambda - \frac{1}{2}J_0(2\sqrt{\alpha\beta})\right\} = 0. \tag{3}$$

From equations (1) and (2), along A_0A_* we have the relation

$$\theta - \alpha - \beta = \frac{1}{4}\pi. \tag{4}$$

Equations (3), (4) determine α, β on A_0A_* as functions of θ. Thus on A_0A_* the dependence of p and φ on θ is determined.

Equations (3), (4) admit the solutions

$$\theta_1 = \pi/4 - \arcsin\varepsilon/2(1-\varepsilon), \quad \alpha_1 = -\arcsin\varepsilon/2(1-\varepsilon), \quad \beta_1 = 0,$$

$$\theta_2 = \pi/4 - \arcsin\varepsilon/2, \quad \alpha_2 = 0, \quad \beta_2 = -\arcsin\varepsilon/2.$$

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Fig. 1

Figure 1: Fig. 1

Differentiating equations (3), (4) and then putting in them $\theta = \theta_2, \alpha = \alpha_2, \beta = \beta_2$, one can obtain the values of the first n derivatives of the functions $\alpha(\theta)$ and $\beta(\theta)$ at the point $\theta = \theta_2$, which makes it possible to write the first $n+1$ terms of the expansion of the functions $\alpha(\theta)$ and $\beta(\theta)$ in a Taylor series in the neighborhood of the point $\theta = \theta_2$. The question of convergence of this series remains open. If ε is given, then one can obtain a solution of equations (3), (4) on an electronic computer.

Denote θ, α, and β at the point A_0 by $\theta_0, \alpha_0, \beta_0$, and at the point A_* by $\theta_*, \alpha_*, \beta_*$. The magnitude of the opening angle of the sector OL_0A_0B is equal to β_0, and of the sector OL_*A_*B is equal to α_*. Therefore the conditions $\alpha_* \leq 0,\ \beta_0 \geq 0$ must be satisfied. Taking into account that in the interval $0 \leq \theta \leq \pi/4$ α and β vanish only once each, and also that $(d\alpha/d\theta)_2 > 0$ and $(d\beta/d\theta)_1 < 0$, we obtain the limits of applicability of the solution found:

$$\theta_0 \leq \theta_1, \qquad \theta_* \leq \theta_2.$$

Fig. 1

The point O must lie on the axis of symmetry of the die. To satisfy this condition, introduce the variables $\bar{x} = s\cos\varphi + t\sin\varphi,\ \bar{y} = -s\sin\varphi + t\cos\varphi$. The quantities $\bar{x}, \bar{y}$ satisfy the equation $\partial^2 f/\partial\alpha\,\partial\beta + f = 0$ (2). Write the equation of the curve A_0A_* in the form $s = \psi_1(\theta),\ t = \psi_2(\theta)$. We have the boundary conditions:

for $s = \psi_1(\theta),\ t = \psi_2(\theta)$

$$\bar{x} = \bar{\psi}_1, \qquad \partial\bar{x}/\partial\alpha = \bar{\psi}_2 + (d\bar{\psi}_1/d\theta - \bar{\psi}_2)/(d\alpha/d\theta), \qquad \partial\bar{x}/\partial\beta = \bar{\psi}_2;$$

$$\bar{y} = \bar{\psi}_2, \qquad \partial\bar{y}/\partial\beta = -\bar{\psi}_1 + (d\bar{\psi}_2/d\theta + \bar{\psi}_1)/(d\beta/d\theta), \qquad \partial\bar{y}/\partial\alpha = -\bar{\psi}_1,$$

for $\alpha = \alpha_0$

$$\bar{x} = 0, \qquad \bar{y} = 0;$$

for $\beta = \beta_*,\ s = a,\ t = b$

$$\partial\bar{x}/\partial\alpha = -a\sin(\alpha + \beta_*) + b\cos(\alpha + \beta_*),$$

$$\partial\bar{y}/\partial\alpha = -a\cos(\alpha + \beta_*) - b\sin(\alpha + \beta_*).$$

Here

$$\bar{\psi}_1 = \psi_1(\theta)\sin(\theta + \pi/4) - \psi_2(\theta)\cos(\theta + \pi/4),$$

$$\bar{\psi}_2 = \psi_1(\theta)\cos(\theta + \pi/4) + \psi_2(\theta)\sin(\theta + \pi/4).$$

Using Riemann' s formula, let us determine $\bar{x}$ and $\bar{y}$ in the region $BDOC$. For $\alpha = \beta = 0$ the condition $\bar{x} - \bar{y} = H_0\sqrt{2}$ must be satisfied. Satisfying it, we obtain the equation

$$\begin{aligned}
&\frac{1}{2}\left(a\sin\theta_* - b\cos\theta_*\right) J_0\!\left(2\sqrt{\alpha_*\beta_*}\right) + \int_{\alpha_*}^{0} [a\cos(\pi/4 + \beta_* + \lambda) + \\
&\qquad + b\sin(\pi/4 + \beta_* + \lambda)]\, J_0\!\left(2\sqrt{\beta_*\lambda}\right)\, d\lambda + \\
&+ \frac{\sqrt{2}}{4}\int_{\theta_0}^{\theta_*}\left\{\left[G(\bar{\psi}_1 + \bar{\psi}_2) + \frac{\partial G}{\partial \alpha_1}(\bar{\psi}_2 - \bar{\psi}_1)\frac{d\alpha_1}{d\theta}\right] + \left[\frac{\partial G}{\partial \beta_1}(\bar{\psi}_1 - \bar{\psi}_2) - G(\bar{\psi}_1 + \bar{\psi}_2)\right]\frac{d\beta_1}{d\theta} + \right. \\
&\qquad \left. + G\left[(\bar{\psi}_1 - \bar{\psi}_2) + \frac{d}{d\theta}(\bar{\psi}_1 + \bar{\psi}_2)\right]\right\} d\theta = H_0.
\end{aligned} \tag{5}$$

Here $G = J_0(2\sqrt{\alpha_1\beta_1})$; $\alpha_1 = \alpha_1(\theta)$ and $\beta_1 = \beta_1(\theta)$ are determined by equations (3), (4).

If the functions $\psi_1(\theta)$, $\psi_2(\theta)$ and the parameters $H_0, \varepsilon, \theta_0, \theta_*$ are given, then equation (5) is satisfied only in exceptional cases. If, however, we have the freedom to choose two parameters in each of the functions $\psi_1(\theta)$ and $\psi_2(\theta)$, then the slip-line field shown in Fig. 1 gives a solution of the problem posed. In this case the values of the parameters mentioned are determined from equation (5) and from the equations $\psi_1(\theta_0) = \psi_2(\theta_0)$, $a - b = H_0\varepsilon\sqrt{2}$. The latter equation requires that the point A_* lie on the straight line $s - t = H_0\varepsilon\sqrt{2}$. For example, there is a one-parameter family of ellipses for which the solution obtained is valid.*

The method presented can be applied to solving the problem of sheet drawing under broader geometrical assumptions, with different laws of friction, and also to solving the problem of rolling.

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REFERENCES

1. B. Druyanov, *Journal of Applied Mechanics and Technical Physics*, No. 6 (1961).

2. R. Hill, *The Mathematical Theory of Plasticity*, 1956.

* An analogous fact holds in the problem of shearing an ideally plastic strip by a punch with a curvilinear base (1).

Note: Figure translations are in progress. See original paper for figures.

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