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Abstract

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PHYSICS

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ON THE KINETIC DESCRIPTION OF QUASI-EQUILIBRIUM TURBULENT PROCESSES IN A PLASMA

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To obtain kinetic equations for a homogeneous plasma with account of polarization, the works of Balescu ([1]), Lenard ([2]), and Silin ([3]) use such solutions of the equations for correlation functions for which the dependence of these functions on time is completely determined by the one-particle distribution functions. Other derivations of the kinetic equation are also possible ([4]). However, the essence of the matter remains the same. This imposes definite restrictions on the type of nonequilibrium processes that can be described by such equations. In particular, these equations are unsuitable for describing unstable states of a plasma.

When the intensity of plasma waves is appreciable, one cannot obtain a closed system of equations for the functions f_a. In the general case one obtains a system of equations for the distribution functions f_a and the distribution function of plasma oscillators ([5]). In the particular case of a homogeneous plasma, which is considered in the present work, one can obtain a simpler system of equations for the functions $f_a(\mathbf{p}, t)$ and the spatial spectral function of the fields $(\delta\mathbf{E}\,\delta\mathbf{E})^u_k$ for the wave numbers at which plasma waves exist.

Such a system of equations makes it possible to describe the change with time of the spectrum of homogeneous turbulence in a plasma. Thus, for example, if, owing to the initial conditions or owing to instability, the energy falling on the plasma waves is considerable, then with the passage of time a redistribution of energy over the spectrum will occur and an equilibrium distribution will be established.

To describe this process, in addition to the equations for the functions $f_a, (\delta\mathbf{E}\,\delta\mathbf{E})^u_k$, it is necessary to know the expressions themselves for the space-time spectral functions, which are also obtained by the method indicated here.

A similar problem was considered in the work of Yu. A. Romanov and G. F. Filippov ([6]). However, the equations obtained there are not complete and therefore cannot serve to describe the redistribution of energy over the spectrum in the establishment of an equilibrium state. In the works of A. A. Vedenov, E. P. Velikhov, and R. Z. Sagdeev ([7]) and ([10]), in essence, two different approximations are considered. One corresponds to the quasilinear approximation for self-consistent equations and has no direct relation to the present work. The second corresponds to the approximation of work ([6]).

As in works ([7,8]), we shall characterize the state of the plasma by the functions $N_a(\mathbf{q},\mathbf{p},t) = \sum_{1\le i\le N_a} \delta(\mathbf{q}-\mathbf{q}_i(t))\delta(\mathbf{p}-\mathbf{p}_i(t))$, where N_a is the number of particles of component a. Denoting by a bar the averaging over the ensemble, we obtain from the equations for N_a, after averaging, the equation for the functions f_a $(\bar{N}_a = n_a f_a,\ n = N_a/V)$

$$\frac{\partial f_a}{\partial t} = -\frac{e_a}{n_a}\frac{\partial}{\partial \mathbf{p}}\overline{(\delta N_a \delta \mathbf{F})}. \tag{1}$$

It follows from this that, to obtain the kinetic equation in this way, one must find an expression for the correlation $\overline{(\delta N_a \delta \mathbf{E})}$ of the functions $\delta N_a = N_a - \overline{N}_a,\ \delta\mathbf{E} = \mathbf{E} - \overline{\mathbf{E}}$.

In the linear approximation the equations for $\delta N_a,\ \delta\mathbf{E}$ have the form

$$\frac{\partial \delta N_a}{\partial t} + \mathbf{v}\frac{\partial \delta N_a}{\partial \mathbf{q}} + e_a n_a \delta\mathbf{E}\,\frac{\partial f_a}{\partial \mathbf{p}} = 0, \tag{2}$$

$$\frac{\partial \delta\mathbf{E}}{\partial t} + 4\pi\sum_a e_a \int \mathbf{v}\,\delta N_a\, d\mathbf{p} = 0. \tag{3}$$

Instead of (3) one may use the equation following from (2), (3),

$$\frac{\partial^2 \delta\mathbf{E}}{\partial t^2} + \omega_L^2 \delta\mathbf{E} = 4\pi \sum_a e_a \int \mathbf{v}\left(\mathbf{v}\frac{\partial}{\partial \mathbf{q}}\right)\delta N_a\, d\mathbf{p}, \qquad \omega_L^2 = \sum_a \frac{4\pi e_a^2 n_a}{m_a}. \tag{4}$$

For spatially homogeneous processes the functions $\overline{\delta N_a(\mathbf{q},\mathbf{p},t)\delta\mathbf{E}(\mathbf{q}',t)}$, $\overline{\delta\mathbf{E}(\mathbf{q},t)\delta\mathbf{E}(\mathbf{q}',t)}$ depend on $\mathbf{q}-\mathbf{q}'$.

Let $(\delta N_a \delta\mathbf{E})k,\ (\delta\mathbf{E}\delta\mathbf{E})k$ denote their Fourier components. A solution is considered in which the spectral functions depend on time only through the functions $f_a(\mathbf{p},t),\ (\delta\mathbf{E}\delta\mathbf{E})^{(\omega)}k$.

The spectral functions are expressed completely in terms of the functions f_a only for values of k for which the conditions

$$\partial f_a/\partial t \ll f_a/\tau_k, \qquad \partial(\delta\mathbf{E}\delta\mathbf{E})_k/\partial t \ll (\delta\mathbf{E}\delta\mathbf{E})_k/\tau_k, \tag{5}$$

are satisfied, where τ_k is the correlation time corresponding to the wave number k. This time increases as k decreases. Let k_{min} be the value of k at which at least one of the conditions (5) is violated. Then the spectral functions are expressed through f_a for $k > k_{\text{min}}$. For $k < k_{\text{min}}$, in addition to the functions f_a, they contain the functions $(\delta\mathbf{E}\delta\mathbf{E})^{(\omega)}k$, for which separate equations are obtained.

The expression for the functions $(\delta N_a(\mathbf{k}\delta\mathbf{E}))k$ has the form (for $\mathbf{kv} \gg \Delta$)

$$e_a(\delta N_a(\mathbf{k}\delta\mathbf{E}))k = i\sum_b \frac{4\pi e_a^2 n_a}{k^2}\, 4\pi e_b^2 n_b \mathbf{k}\,\frac{\partial f_a}{\partial \mathbf{p}} \int \frac{\dot{f}_b(\mathbf{p}')\,d\mathbf{p}'}{(\mathbf{kv}-\mathbf{kv}'-i\Delta)\,|\varepsilon(\mathbf{kv},k)|^2} +$$

$$+\, i4\pi e_a^2 n_a \mathbf{k}\frac{\partial f_a}{\partial \mathbf{p}} \int \frac{(\omega')^{-1} B(\omega',k)d\omega'}{\mathbf{kv}-\omega'-i\Delta}\, \frac{(\delta\mathbf{E}\delta\mathbf{E})k}{4\pi} + i4\pi e_a^2 n_a\, \frac{f_a}{\varepsilon^{(-)}(\mathbf{kv},k)}. \tag{6}$$

Here

$$\varepsilon^{(\pm)}(\omega,\mathbf{k}) = \varepsilon' + i\varepsilon'' = 1 + \sum_a \frac{4\pi e_a^2 n_a}{k^2} \int \frac{\mathbf{k}\,\partial f_a/\partial \mathbf{P}}{\omega - \mathbf{kv} \pm i\Delta}\, d\mathbf{p}$$

is the dielectric permittivity.

The quantity Δ in (6) satisfies $1/\tau_{k_{\text{min}}} \ll \Delta \ll \omega_L$, and the function

$$B(\omega,\mathbf{k}) = \Delta \frac{\partial}{\partial\omega}\varepsilon'(\omega,\mathbf{k}) \Big/ \pi\left[\left(\varepsilon' - \Delta\frac{\partial \varepsilon''}{\partial\omega}\right)^2 + \left(\varepsilon'' + \Delta\frac{\partial\varepsilon'}{\partial\omega}\right)^2\right]. \tag{7}$$

For $\varepsilon'' \ll \varepsilon'$ (or for small k),

$$B(\omega,\mathbf{k}) = \text{sing}\left[\frac{\partial}{\partial\omega}\varepsilon'(\omega,\mathbf{k})\right]\delta(\varepsilon'(\omega,\mathbf{k})) \quad \text{as } \Delta \to 0. \tag{8}$$

In all final formulas $\Delta \to 0$. This means that $1/\tau_{k_{\text{min}}}\omega_L$ is neglected in comparison with unity.

Substituting expression (6) into (1), we obtain the equations for f_a

$$\frac{\partial f_a}{\partial t} = \frac{\partial}{\partial p_i} D_{ij}^a \frac{\partial f_a}{\partial p_j} + \frac{\partial}{\partial p_i}\left(A_i^a f_a\right). \tag{9}$$

Here

$$D_{ij}^a = \sum_b 2e_a^2 e_b^2 n_b \int \frac{k_i k_j}{k^4} \frac{\delta(\mathbf{kv} - \mathbf{kv}')}{|\varepsilon(\mathbf{kv}, \mathbf{k})|^2} f_b(p')\, d\mathbf{k}\, d\mathbf{p}' +$$

$$+ \left(\frac{e_a^2}{2\pi} \int (\mathbf{kv})^{-1} B(\mathbf{kv}, \mathbf{k}) \frac{k_i k_j}{k^2} \frac{(\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k}}^{(u)}}{4\pi}\, d\mathbf{k} + \frac{e_a^2}{2\pi\omega_L^2} \int \frac{k_i k_j}{k^2} P_k\, d\mathbf{k} \right), \quad (10)$$

$$A_i^a = -\sum_a 2e_a^2 e_b^2 n_b \int \frac{k_i k_j}{k^4} \frac{\delta(\mathbf{kv} - \mathbf{kv}')}{|\varepsilon(\mathbf{kv}, \mathbf{k})|^2} \frac{\partial f_a}{\partial p_j'}\, d\mathbf{k}\, d\mathbf{p}' +$$

$$+ \frac{e_a^2}{2\pi} \int \frac{k_i}{k^2} B(\mathbf{kv}, \mathbf{k})\, d\mathbf{k}. \quad (11)$$

The coefficients (10), (11) can be represented in the form of the sums $D_{ij} = D_{ij}^{(cm)} + D_{ij}^{(u)}$, $A_i = A_i^{(cm)} + A_i^{(u)}$ of two terms, the first of which describe the effect of "collisions" of charged particles with allowance for polarization, while the second are due to the effect of emission of plasma waves. In the expressions $D_{ij}^{(cm)}$, $A_i^{(cm)}$, the integration is carried out over the region $k > k_{\min}$.

Equations (9) coincide with the kinetic equations obtained in papers ([1],[2]), if $D_{ij}^{(u)} = 0$, $A_i^{(u)} = 0$, i.e., if the emission of plasma waves is insignificant and only "collisions" are taken into account.

Equations (9), (13) are written here for the case when the roots $\varepsilon'(\omega, k) = 0$ are close to ω_L.

In order to obtain a closed system of equations, one must also write an equation for the functions $(\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k}}^{(u)}$.

For this purpose, using equation (3), we write the equation

$$\frac{\partial}{\partial t} (\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k}} = -8\pi\ \mathrm{Re}(\delta\mathbf{j}\delta\mathbf{E})_{\mathbf{k}} = -8\pi \sum_a e_a \int \frac{(\mathbf{kv})}{k^2} \mathrm{Re}(\delta N_a (\mathbf{k}\delta\mathbf{E})_{\mathbf{k}}\, d\mathbf{p}. \quad (12)$$

Using expressions (6) and (2), (3) for $\mathbf{kv} \ll \Delta$, we hence find the desired equation

$$\frac{\partial}{\partial t} (\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k}}^{(u)} = \pi \sum_a \frac{(4\pi)^2 e_a^2 n_a}{k^2} \int B(\mathbf{kv}, \mathbf{k}) \left\{ \frac{(\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k}}^{(u)}}{4\pi} \mathbf{k} \frac{\partial f_a}{\partial \mathbf{p}} + \right.$$

$$\left. + (\mathbf{kv}) f_a \right\} d\mathbf{p} = 8\pi P_k. \quad (13)$$

The function $B(\omega, \mathbf{k})$ in (13) is determined by expressions (7), (8). Equations (9), (13) also constitute a closed system of equations for the functions f_a, $(\delta\mathbf{E}\delta\mathbf{E})^{(u)}_{\mathbf{k}}$.

This system of equations describes the temporal change of homogeneous turbulence in a plasma. At the same time, if, owing to the initial conditions or as a result of instability, the energy going into plasma waves is significant, i.e., the function $(\delta\mathbf{E}\delta\mathbf{E})^{(u)}_{\mathbf{k}}$ is appreciable, then in the course of time a redistribution of energy over the spectrum will occur. As a result, a Maxwellian distribution over velocities will be established, and the correlations will be determined by the Debye correlation function. To show this, we give the expression for the complete space-time spectral function of the fields

$$(\delta\mathbf{E}\delta\mathbf{E})_{\omega,\mathbf{k}} = \sum_a \frac{(4\pi)^2 e_a^2 n_a}{k^2} \int \frac{2\pi\delta(\omega - \mathbf{k}\mathbf{v}) f_a}{|\varepsilon(\omega,\mathbf{k})|^2}\, d\mathbf{p} + 2\pi \frac{B(\omega,\mathbf{k})}{\omega} (\delta\mathbf{E}\delta\mathbf{E})^{(u)}_{\mathbf{k}}. \quad (14)$$

In the equilibrium case

$$\frac{(\delta\mathbf{E}\,\delta\mathbf{E})^{(u)}_{\mathbf{k}}}{4\pi} = \varkappa T,$$

but

$$\frac{(\delta\mathbf{E}\,\delta\mathbf{E})_{\mathbf{k}}}{4\pi} \equiv \frac{1}{8\pi^2}\int_{-\infty}^{\infty} (\delta\mathbf{E}\,\delta\mathbf{E})_{\omega,\mathbf{k}}\, d\omega = \frac{\varkappa T}{1 + r_d^2 k^2}.$$

Thus, in the nonequilibrium case, the space-time spectral functions likewise cannot be expressed completely in terms of the one-particle distribution functions f_a. As in the other formulas, the first term in (14) pertains to the region $k > k_{\min}$.

Using the results of the present work, one can also determine more general space-time correlation functions. They will also be valid for describing unstable states for which the growth of $(\delta\mathbf{E}\,\delta\mathbf{E})^{(u)}_{\mathbf{k}}$ is limited by changes in the functions f.

The equations obtained here can be generalized to the case of weak inhomogeneity of the plasma and used to derive hydrodynamic equations that take into account, in the dissipative terms, the radiation of plasma waves.

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Note: Figure translations are in progress. See original paper for figures.