STATISTICAL DESCRIPTION OF A TURBULENT JET

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Abstract

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The physical model of mechanical mixing (diffusion) of “cold”and “hot”volumes of a turbulent nonisothermal jet can be described by the equations of conservation of volumes, conservation of the momentum of volumes, and Taylor’ s equation. Taylor’ s equation, generalized to the case of inhomogeneous turbulence (along the flow) and averaged over the mixing zone (across the flow), can be written in the form:

$$\frac{d\sigma^2}{2dx} \simeq \frac{D_0 + D_{\mathrm{T}}(x)}{\bar{u}_{\mathrm{cp}}(x)}; \tag{1}$$

$D_0, D_{\mathrm{T}}(x)$ are the coefficients of turbulent diffusion, determined by the initial turbulence and by the turbulence of the jet; $\sigma^2(x)$, $\bar{u}_{\mathrm{cp}}(x)$ are the variance and mean velocity of the volumes in the mixing zone of the two streams.

For a submerged axisymmetric jet, according to thermoanemometric measurements by Corrsin (1) and Laurence (2), $v' \sim \bar{u}_{\mathrm{cp}} \sim 1/x$, $l_{\mathrm{T}} \sim x$, as a result of which the coefficient of turbulent diffusion proves to be approximately constant along the jet, and the quantity $\sigma(x)$ varies practically linearly (v', l_{T} are the turbulent velocity and the scale of turbulence).

In a real turbulent jet, at the instantaneous interface of two media, discontinuities may occur not only in density (temperature), but also in the mean velocities, selectively averaged either only over the cold volumes or only over the “hot” ones. In the present case, as a working hypothesis, we adopt the condition that there is no discontinuity of the mean velocities. In physical meaning this hypothesis corresponds to the case of a strongly curved and entangled interface of the two media, when in the folds of this surface the more slowly moving medium is completely entrained by the volumes of the medium moving with the greater velocity.

Let us consider the outflow of a jet from a circular orifice of diameter $2a_0$ into a coflowing stream. The initial velocity, density, and temperature of the jet are v_2, ρ_2, and T_2; the corresponding parameters of the coflowing stream are v_1, ρ_1, and T_1.

If P_2 is the probability of detecting, at a given point, the substance of the jet, and P_1 is the probability of detecting the substance of the coflowing stream, the

Fig. 1. Fields of excess temperatures. I—across the jet, II—along the jet. Processing of Pabst' s data

Figure 1: Fig. 1. Fields of excess temperatures. I—across the jet, II—along the jet. Processing of Pabst' s data

following relations hold:

$$P_1 + P_2 = 1; \qquad \bar{\rho} = \rho_1 P_1 + \rho_2 P_2; \qquad \bar{T} = T_1 P_1 + T_2 P_2; \tag{2}$$

$$\bar{u} = \bar{u}_1 P_1 + \bar{u}_2 P_2 = \bar{u}(P_1 + P_2) = \bar{u},$$

since $\bar{u} = \bar{u}_1 = \bar{u}_2$, where $\bar{\rho}, \bar{T}, \bar{u}$ are the mean values of density, temperature, and velocity.

The equations of turbulent transport of volumes and of the momentum of volumes, written in cylindrical coordinates, have the form (longi-

turbulent diffusion and the pressure drop are neglected):

$$\frac{1}{r}\frac{\partial}{\partial r}(r\bar{v}P_2) + \frac{\partial \bar{u}P_2}{\partial z} = D_T \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P_2}{\partial r}\right), \tag{3a}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\bar{v}P_1) + \frac{\partial \bar{u}P_1}{\partial z} = D_T \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P_1}{\partial r}\right), \qquad (3\ )$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\bar{j}\bar{v}) + \frac{\partial j\bar{u}}{\partial z} = D_j \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \bar{j}}{\partial r}\right), \qquad (3\ )$$

where $\bar{v}$ is the radial component of the mean velocity; $\bar{u}$ is the component directed along the polar axis z, coinciding with the axis of the jet; $\bar{j}$ is the longitudinal component of the mean momentum per unit volume; D_T, D_j are the coefficients of turbulent diffusion of volume and of momentum per unit volume.

Fig. 1. Fields of excess temperatures. I—across the jet, II—along the jet. Processing of Pabst' s data

A rigorous mathematical justification of equations such as (3), under certain assumptions (a process without aftereffect, which is equivalent to an approximate writing of Taylor' s equation ([1]), etc.), was given in the work of A. N. Kolmogorov ([3]).

The sum of (3a) and (3) gives the generalized continuity equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\bar{v}) + \frac{\partial \bar{u}}{\partial z} = 0. \tag{4}$$

Figure 2

Figure 2: Figure 2

From (3) there also follow the laws of constancy of the mean fluxes of the jet mass and of the total momentum:

$$\int_0^\infty \bar{u} P_2 r\, dr = \frac{v_2 a_0^2}{2}; \qquad \int_0^\infty \bar{j}\bar{u} r\, dr = \frac{a_0^2 v_2^2 \rho_2}{2} + S_1, \tag{3'}$$

where S_1 is the initial momentum flux of the accompanying stream.

We seek $\bar{u}$ and P_2 in the following form:

$$\bar{u}(r,z) = \frac{v_2 - v_1}{\sigma_v^2} \int_0^{a_v} e^{-(r^2+r_1^2)/2\sigma_v^2} J_0\left(\frac{irr_1}{\sigma_v^2}\right) r_1\, dr_1 + v_1,$$

$$P_2(r,z) = \frac{\bar{T} - T_1}{T_2 - T_1} = \frac{1}{\sigma_T^2} \int_0^{a_T} e^{-(r^2+r_1^2)/2\sigma_T^2} J_0\left(\frac{irr_1}{\sigma_T^2}\right) r_1\, dr_1, \tag{5}$$

where $a_v(z)$ and $a_T(z)$ are the mean radii (velocity and temperature) of the jet, determined by the relations

$$\int_0^\infty \frac{\bar{u} - v_1}{v_2 - v_1} r\, dr = \frac{a_v^2}{2}; \qquad \int_0^\infty P_2 r\, dr = \frac{a_T^2}{2}. \tag{6}$$

The parameters $\sigma_v(z)$ and $\sigma_T(z)$ describe the variance of the initial profiles of velocity and temperature due to turbulent diffusion (they are determined by specifying the values of D_T, D_j).

Fig. 2. Variation of the temperature and velocity radii of an isothermal jet in a coflow, for $D_v/D_T = 0.5$ ($m = V_1/V_2$)

With an appropriate choice of the parameters D_j, D_T, constant along the jet, the theoretical temperature and velocity profiles (5) coincide with the experimental ones (Fig. 1).

Let us determine the relation of the mean radii a_v, a_T to the variance of the jet. For $a_v \ll \sigma_v$ and $a_T \ll \sigma_T$, instead of (5) we may write:

$$\bar{u} = \frac{v_2 - v_1}{2\sigma_v^2} a_v^2 e^{-r^2/2\sigma_v^2} + v_1,$$

$$P_2 = \frac{a_T^2}{2\sigma_T^2} e^{-r^2/2\sigma_T^2}. \tag{7}$$

Figure 2

Figure 3: Figure 2

Taking into account that, by virtue of (2), $\bar{j} = \bar{\rho}\bar{u} = \bar{\rho}\,\bar{u}$, and substituting (7) into (3′), we obtain ($n = \rho_1/\rho_2$, $m = v_1/v_2$)

$$(1-m)\frac{a_v^2 a_T^2}{2(\sigma_v^2+\sigma_T^2)} + m a_T^2 = a_0^2,$$

$$n(1-m)^2\frac{a_v^4}{8\sigma_v^2} + (1-n)(1-$$

$$-m)^2\frac{a_v^4 a_T^2}{8(2\sigma_T^2+\sigma_v^2)\sigma_v^2}+ \tag{8}$$

$$+nm(1-m)a_v^2 + (1-$$

$$-n)m(1-m)\frac{a_v^2 a_T^2}{2(\sigma_v^2+\sigma_T^2)}+$$

$$+\frac{nm^2 a_0^2}{2} + \frac{(1-n)m^2 a_T^2}{2} = \frac{a_0^2}{2}.$$

Fig. 2. Variation of the temperature and velocity radii of a flooded jet at various superheats and for $D_v/D_T = 0.5$ ($n = \rho_1/\rho_2 = T_2/T_1$)

If the relation between σ_v and σ_T is known, relations (8) make it possible to give a complete description of the flow with the aid of a single parameter σ_T. Below it is assumed, in accordance with the available experimental data, that $\sigma_v^2 \simeq \frac{1}{2}\sigma_T^2$.

In an isothermal flooded jet ($m = 0$, $n = 1$), the momentum is proportional to the momentum-velocity D_j; one may write, instead of the diffusion coefficient D_v, the velocity-diffusion coefficient. Let us write Taylor’ s equation for

temperatures and velocities:

$$\frac{D_T}{\bar{u}_{T\mathrm{cp}}} = \frac{1}{2}\frac{d\sigma_T^2}{dz}, \qquad \frac{D_v}{\bar{u}_{v\mathrm{cp}}} = \frac{1}{2}\frac{d\sigma_v^2}{dz}, \tag{9}$$

where $\bar{u}_{T\mathrm{cp}}$ and $\bar{u}_{v\mathrm{cp}}$ are certain characteristic velocities.

Using the mathematical relations

$$\frac{\partial P_2}{\partial z} = \frac{\partial P_2}{\partial a_T}\frac{da_T}{dz} + \frac{\partial P_2}{\partial \sigma_T^2}\frac{d\sigma_T^2}{dz}, \qquad \frac{\partial P_2}{\partial \sigma_T^2} = \frac{1}{2r}\frac{\partial}{\partial r}\left(r\frac{\partial P_2}{\partial r}\right),$$

we obtain, for $r = 0$, from equations (3):

$$\frac{da_T}{dz} - \left(1 - \frac{\bar{u}_{T\text{cp}}}{u_m}\right)\frac{a_T}{2\sigma_T^2}\frac{d\sigma_T^2}{dz} = 0, \qquad \frac{da_v}{dz} - \left(1 - \frac{\bar{u}_{v\text{cp}}}{u_m}\right)\frac{a_v}{2\sigma_v^2}\frac{d\sigma_v^2}{dz} = 0, \quad (10)$$

where $\bar{u}_m$ is the velocity on the jet axis. In deriving (10) it was taken into account that

$$\bar{v}(0,z) = 0, \qquad \bar{u}_m = v_2\left(1 - e^{-a_v^2/2\sigma_v^2}\right), \qquad P_2(0,z) = 1 - e^{-a_T^2/2\sigma_T^2}. \quad (11)$$

By virtue of (8), it follows from (10) that

$$\bar{v}_{T\text{cp}} = \bar{u}_{v\text{cp}} = \bar{u}_m/2. \quad (12)$$

The distribution of the transverse velocity, by virtue of (4), is given by the formula

$$\bar{v}(r) = -\frac{V_2}{2r}\frac{d}{dz}\left[a_v^2\left(1 - e^{-r^2/2\sigma_v^2}\right)\right]. \quad (13)$$

Let us also note several obvious relations for an isothermal submerged jet, following from (8):

$$a_T = \sqrt{\frac{3}{2}}\,a_v, \qquad P_2(0,z) = \frac{3}{4}\frac{u_m}{V_2},$$

which agree well with experimental data.

In Figs. 2 and 3 the mean radii of a nonisothermal coflowing jet, calculated by formula (8), are plotted; here σ_v was determined from experiments on jets in a coflowing stream ([4]) by the conversion $\sigma_v = 0.85\bar{r}_c$, where $\bar{r}_c$ is found from the condition (see (7)) $(\bar{u} - V_1)/(\bar{u}_m - V_1) = 0.5$.

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Note: Figure translations are in progress. See original paper for figures.